Unit 4: Linear Functions
(6 Weeks)

UNIT OVERVIEW

Students start Unit 4 by exploring the distinction between linear and nonlinear behavior, and then focus on learning about linear functions. Throughout Unit 4, students derive linear models of real-world situations in order to analyze situations, make predictions or solve problems. Analyzing situations often takes the form of identifying the real world meaning of the slope and the x- and y-intercepts of a linear model. Making predictions involves evaluating models for a given independent variable (given x find y), and solving equations for the independent variable given the dependent variable (given y find x). Problem solving occurs through the use of various representations: algebraic, tabular, graphic and numeric.

In the first investigation students begin to develop the concept of constant rate of change by examining the data generated by a motion detector as displayed in a time-distance graph. Students will understand that “walking steadily” creates a straight-line graph, whereas “speeding up” or “slowing down” creates a graph that is non-linear. Decreasing the distance from the starting place (the motion detector) will produce a graph that decreases as one reads from left to right. Conversely, increasing the distance from the starting place will create a graph that increases as one reads from left to right.

Investigation 2 provides practice recognizing and describing functions from data tables and graphs in various contextual situations. Students identify the characteristics of a linear function, investigate the role of slopes and y-intercepts in the graphs of functions and relate this information to the context of various problems. Students create graphs by hand and with the graphing calculator. They engage in activities that highlight the capability of linear functions to model a wide range of real world relationships. A quiz is suggested after this investigation.

The third investigation provides an in-depth focus on slope as constant additive change, a definitive attribute of linear functions. Students calculate the slope from data in tables and graphs. They identify and interpret the slope from real-world linear situations as the constant rate of change in the dependent variable compared to the change in the independent variable.

The fourth investigation fully develops the \( f(x) = mx + b \) and \( y = mx + b \) forms, which students intuitively used to model a variety of real world situations and to define patterns explicitly as far back as Unit 1. Students explore the results of how changing the two parameters \( m \) and \( b \) changes the graph of a linear function. They will discover that changing the y-intercept causes a vertical shift in the graph, that the sign of the slope determines whether the graph is increasing or decreasing, and that the magnitude of the slope affects the steepness of the graph. Students will be able to graph a function given in slope-intercept form not only by making a table of values, but also by first plotting the y-intercept and then one or more additional points using the slope. Students will be able to find the slope-intercept equation of a line from a graph, table or real-world scenario, thus reinforcing the multi-representational approach. As in other investigations, students have opportunities to use what they are learning to solve a variety of
contextual problems. To complete the investigation students will discover the relationships of the slopes of parallel lines and of perpendicular lines. At this point, students may be assessed by a Mid-Unit Test.

Investigation 5 introduces the standard form of the equation of a line. Equations in standard form are graphed two ways: by finding the x- and y-intercepts and by transforming the standard equation to slope-intercept form. Students will also investigate direct variation problems, which are modeled by the family of linear functions that have a y-intercept of 0. A quiz is suggested after this investigation.

The sixth investigation develops the point-slope form of the line. Students can identify a point and the slope from the equation and can write an equation in point-slope form given a point and slope or given two points. Students transform equations written in point-slope form into slope-intercept or standard form. Contextual activities and real world situations are used to introduce the forms of linear equations as well as to extend students’ knowledge of linear functions. By the end of this investigation, students will choose which of the three forms of linear equations is most advantageous for solving a particular problem.

The performance task assesses the ability of students to distinguish between linear and nonlinear relationships and then make predictions based on the linear functions. The End-of-Unit Assessment checks for mastery of the key concepts and skills in the unit.

**Essential Questions**
- What is a linear function?
- What are the different ways that linear functions may be represented?
- What is the significance of a linear function’s slope and y-intercept?
- How may linear functions model real world situations?
- How may linear functions help us analyze real world situations and solve practical problems?

**Enduring Understandings**
Linear functions are characterized by a constant average rate of change (or constant additive change).

**Unit Contents**
- Investigation 1: What Makes a Function Linear? (2 days)
- Investigation 2: Recognizing Linear Functions from Words, Tables and Graphs (4 days including Quiz on Investigations 1 and 2)
- Investigation 3: Calculating and Interpreting Slope (4 days)
- Investigation 4: Effects of Changing Parameters of an Equation in Slope-Intercept Form (4 days)
- Mid-Unit Test (2 days including review)
- Investigation 5: Forms of Linear Equation (4 days including Quiz on Investigation 5)
- Investigation 6: Point-Slope Form of Linear Equations (4 days)
- Performance Task: Linear Models (2 days)
- End of Unit Test (2 days including review)
Common Core Standards

Mathematical Practices #1 and #3 describe a classroom environment that encourages thinking mathematically and are critical for quality teaching and learning. Practices in bold are to be emphasized in the unit.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards Overview

- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations
- Construct and compare linear [and exponential] models and solve problems
- Interpret expressions for functions in terms of the situation they model

Standards with Priority Standards in Bold

F-IF 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

F-IF 7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
   a. Graph linear ...functions and show intercepts..

F-IF 8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F-LE 1. Distinguish between situations that can be modeled with linear functions [and with exponential functions].
   a. Prove that linear functions grow by equal differences over equal intervals...
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another....

F-LE 2. Construct linear ... functions, including arithmetic ... sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F-LE 5. Interpret the parameters in a linear ... function in terms of a context.
### Vocabulary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tr>
<td>Constant Additive Change</td>
<td>Linear Models</td>
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<tr>
<td>Convex Polygon</td>
<td>Magnitude</td>
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<td>Dependent Variable</td>
<td>Nonlinear Function</td>
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<td>Direct Variation</td>
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<td>Independent Variable</td>
<td>Piecewise Function</td>
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<td>Initial Value</td>
<td>Point-Slope Form</td>
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<td>Linear Function</td>
<td>Rate of Change</td>
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<td>Slope</td>
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<td>Magnitude</td>
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<td>Piecewise Function</td>
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<td>Point-Slope Form</td>
<td>x-intercept</td>
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<td>Rate of Change</td>
<td>y-intercept</td>
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### Assessment Strategies

**Performance Task: Linear Models**

The Unit 4 performance task *Linear Models* requires students to investigate population growth or population decline. Students will be expected to demonstrate and apply their understanding of key concepts related to linear functions. Students will examine and compare the accuracy of their linear population models and present their findings in class.

**Other Evidence (Formative and Summative Assessments)**

- Exit slips
- Class work
- Homework assignments
- Math journals
- Unit 4 Investigations 1 & 2 Quiz
- Mid-Unit Test
- Unit 4 Investigation 5 Quiz
- Unit 4 Test
Unit 4 Materials List

Investigation 1

- Motion detector such as those by Vernier, together with either (a) a calculator with view screen to project to class OR (b) a computer that projects to entire class. The calculator software is under the APPS key. You may see CBL/CBR or Ranger, for example. You can also download programs like RANGER, HIKER or DATA MATCH from the website [www.Education.TI.com](http://www.Education.TI.com). The software for the computer called LOGGER LITE is free on the Vernier website [http://www.vernier.com/products/software/logger-lite/](http://www.vernier.com/products/software/logger-lite/). To connect the motion detector to the computer, you will need a printer cable commonly used for many printers. The same printer cable is also sold as the Go Motion cable at Vernier.

- Timer, tape measure and chalk to mark floor location as person walks

- [http://illuminations.nctm.org/LessonDetail.aspx?ID=U188](http://illuminations.nctm.org/LessonDetail.aspx?ID=U188) for an NCTM Illuminations lesson entitled “Movement with Functions”

- Time-Distance lessons under “Classroom Activities” on the Texas Instruments website: [http://education.ti.com/calculators/downloads/US/Activities/Search/Keywords?k=time+distance+graphs](http://education.ti.com/calculators/downloads/US/Activities/Search/Keywords?k=time+distance+graphs)

- Workbooks from Texas Instruments such as activities 1 and 13 in *Real World Math Made Easy* by Chris Brueningsen et. al. or *CBR Explorations: Math and Science in Motion* by Brueningsen et. al. or *CBR Explorations: Modeling Motion: High School Activities with the CBR* by Linda Antinone et. al.

Investigation 2

- Straightedges for drawing linear graphs

- Graphing calculators

Investigation 3

- Straightedges for drawing linear graphs

- Graphing calculators

Investigation 4

- Straightedges for drawing linear graphs

- Graphing calculators
Investigation 5

- Rulers
- Graphing Calculators
- Slope intercept game links: http://hotmath.com/hotmath_help/games/kp/kp_hotmath_sound.swf
- Video on roof trusses http://www.youtube.com/watch?v=FbQxN7iv-ns&feature=related

Investigation 6

- Rulers
- Forest Elementary Students: http://www.hometownlife.com/article/20120603/NEWS06/206030372/Forest-students-recycle-water-bottles-make-greenhouse
## CT Algebra I Model Curriculum

### Web Sites for Unit 4

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<tr>
<th>Where used</th>
<th>Web site address</th>
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Unit 4: Investigation 1 (2 Days)

WHAT MAKES A FUNCTION LINEAR?

CCSS: FLE-1, FIF-7A

Overview
Students distinguish non-linear functions from linear functions by exploring distance as a function of time in verbal, graphical and tabular form. Students learn that linear functions are characterized by a constant rate of change.

Assessment Activities

Evidence of Success: What Will Students Be Able to Do?
• Interpret distance-time graphs and tables in terms of the motion of an object.
• Write a verbal description of a distance-time function, sketch its graph, and construct a table of values.
• Distinguish between linear and non-linear functions by recognizing that linear functions have a constant rate of change whether the function is given verbally, graphically, or in table form. (Note: Calculation of slopes will be developed more fully in Investigation 3.)
• Identify distance-time functions with slopes of different magnitudes from the verbal description, the graph, and the table.
• Distinguish between distance-time functions with positive slopes (increasing functions) and functions with negative slopes (decreasing functions) given a verbal, graphical or tabular representation of the function.

Assessment Strategies: How Will They Show What They Know?
Exit Slip 4.1 asks students to draw qualitative graphs of someone walking with the motion detector.
Exit Slip 4.1.2 asks students to draw quantitative graphs of someone walking with the motion detector given speed as well as direction.
Journal entry asks students to draw a distance-time graph based on a story.

Launch Notes
The theme of this investigation is motion. To stimulate student interest in the activity, you may play the video of Carole King singing her classic song “Do the Locomotion” with Slash in concert as students enter class. The video is located at http://www.youtube.com/watch?v=ehVaMey2e7A&feature=related. Locomotion refers to how animals, including people, move.

Ask the students to guess what today’s lesson is about as you show any of the several videos available on the internet about how fast and/or how slowly animals move. A quick internet search for “fastest animals” videos yielded:
Choose your motion video according to your students’ interests or the interests that you want to encourage among your students. Next, challenge students to find how fast a person walks on average, leading them to understand the units of speed to be units of distance divided by units of time. To test their hypotheses, show them how to create a distance-time graph first by gathering data by hand, then by using a motion detector. Be sure to display tables and graphs of a variety of walks: slow, fast, forward, backward, constant rate, standing still, slowing down, or speeding up. The students will analyze the relationship between the graph, the tables, and the verbal description of the person’s motion. During the lesson you can test the students’ hypotheses about how fast a person walks, on average, by looking at the time-distance table and by observing the steepness of the distance-time graph.

**Closure Notes**

This investigation culminates with students being able to produce graphs and descriptions for distance-time functions. Students will identify that a constant rate of change creates a linear graph. Students will understand that steepness of a linear graph is determined by the speed of the object. They will identify increasing, decreasing and constant lines as corresponding to objects that move away from the motion detector, come toward the motion detector, or stand still, respectively.

**Teaching Strategies**

I. Engage the class in a discussion about the velocity of people, animals, or the objects depicted in the launch videos. You may inform the class that they are going to be studying the velocity of their own walk during these activities. The magnitude and direction of velocity correspond to the two independent questions you can ask the students: how fast is the object moving, and in what direction is the object moving? Slope need not be defined yet, because Investigation 3 will formalize the concept of slope.

Depending on the needs of your class, you can start with the motion detector lesson right away, or you can first have students gather and plot distance-time data by hand before you show them the motion detector. To demonstrate how to measure and display distance as a function of time without a motion detector, have one student walk very slowly and at a constant rate in a line along a tape measure spread on the floor or along chalk marks that indicate feet or meters on the floor. Have another student call out “time” every two or three seconds, and
have a third person observe or mark with a chalk the position of the walker along the measured line. Record the time and distance data in a table, using function vocabulary students learned in unit 3. Then describe the person’s walk, and create a distance-time graph by hand. Ask students to determine, on average, how far the person walks in 6 seconds, and in 1 second.

To use the motion detector you will need either a computer or graphing calculator with classroom display so that the class can see the motion graph of a person walking. Explain to the students that the motion detector works using the principles of sonar, so they will hear the detector emit a series of clicking sounds that will then bounce off the object it points toward. The equipment will measure how much time elapses between emitting the sound and receiving the sound echo.

Draw set of axes for a first quadrant graph on the board. Let time be represented on the x-axis and distance from the motion detector on the y-axis. Ask for a volunteer and have the student stand at the motion detector and walk away from it when you tell them to start. Trace the path created by the volunteer and the x- and y-axes on the board. Ask the class if someone can walk a steeper line and have a volunteer try. Trace this path on the same set of axes. Ask the class if someone can walk a less steep line and have a volunteer try. Trace this path. Ask the class what determined how steep the line was? If the path is a line, then ask what about the person’s walk created the line. If it isn’t a line, then ask why not and what might we change so that the path is a line.

Now collect data from a variety of walks, encouraging the class to ask creative questions, test their hypotheses, and generally explore distance-time graphs with the motion detector. You may lead the discussion by asking the following questions and allowing students opportunities to imitate certain motions.

- How could a walker create a line that slants from the bottom left to the top right?
- How could a walker create a line that slants from the top left to the bottom right?
- Why do we talk about the line going from left to right?
- How they would create a steeper line or a less steep line?
- What determines the steepness of the line?
- What does it take to walk a straight line?
- What is the domain and range from the graph?
- Can a person walk a wave?

Have the students calculate how fast one of the students walked by using the table of values or by using the trace feature on the graph of the line to display coordinates of two points.

II. **Activity 4.1.1 Motion Graphs – What Makes a Function Linear?**, students predict what distance-time graphs will look like before collecting distance-time
data using a motion detector. Students can use a motion detector to verify that they have the correct graphs in question 3. This activity can be completed without using a motion detector. By the end of the activity, students should have concluded that a person walking at a steady pace will produce a graph of a line; faster walks produce steeper lines; walking forward creates an increasing function and backwards a decreasing one, standing still produces a horizontal line; and from the table you can determine the average velocity of the person walking. 

**Exit Slip 4.1.1** and **Activity 4.1.2 Motion Graph Scenarios** can be assigned in class or for homework to reinforce and assess student understanding.

**Alternative Activity If Motion Detectors Are Not Available:** If you do not have a motion detector, you may want to discuss materials and data collection ideas with the physics teacher at your school. Then you can adapt the problems from the motion detector part of this investigation by having students roll a ball or toy car at different speeds along a paper tape on a table, marking the position of the object every few seconds. For example, one could lay a strip of paper (such as a cash register roll of paper) alongside the path of something that moves such as a rolling ball or a toy car. As the object moves along the paper tape, one person marks the position of the object with a dot as the timer calls out the seconds. The disadvantage of this method is that linear data is not possible since the ball or toy car will slow down unless you have a motorized toy.

Another way to collect distance-time data is to fix the distances (the dependent variable) and record the time (the independent variable) as a person or toy reaches a certain distance marker. One could mark distances on a floor or table with a tape measure, and then record the time when the object reaches every 6 inches or 10 cm. Observe what happens to the distance between dots as the object speeds up or slows down. Note that dot pattern of objects moving at constant velocity will give equally spaced dots; and the distance between dots divided by the interval of time gives the velocity of the object. An object that accelerates will have dots further and further apart, and an object that slows down will have dots closer together. A strobe photograph of a drip coming out of a faucet gives analogous information about the speed of the falling drip.

### III.

**Activity 4.1.3 More Motion Graphs** provides students additional practice with distance-time functions. As students complete this activity, you may use Data Match, a built-in application on the Logger Lite and the TI-calculator motion detector programs. Data Match allows students to attempt to walk in such a way that produces a graph identical to one displayed on the screen. Students should be looking at the screen as they produce their walks so that they can make adjustments during their walk. The graphs in Data Match are piecewise linear functions. When using Data Match, prompt each student group to first describe the walk to be matched, present their description to the class for discussion, and then send a representative to the front to test out their idea with the motion
detector in front of the class. As you circulate among the groups, elicit answers to the following sample questions:

- Where is walker going to start?
- What do we know about the distance and time from the graph?
- What is the walker going to do? What’s her plan?
- How fast should she walk?
- How could we figure out exactly how fast she should walk?
- What is a situation in real life that could be modeled by a graph that looks like this?

Whole class discussions can be lively as students argue for and refine their ideas about how to describe each walk for the data matches. Assess whether students are able to describe the underlying mathematical concepts of constant rate of change or describe the magnitude and direction of the rate of change from data tables and graphs.

Ask the class if they could determine the speed at which the person walked and if so, how. Using the table feature, display the data points that correspond with the last graph created and have the students calculate the walker’s rate. Some students may make the connection that they are actually calculating the slope. Explain that slope is the rate at which distance changes as time changes. Pay attention to the units of measure of rate: meters per second, or feet per second. Connect the idea to the units on the vertical and the horizontal axis of the distance time graph. Ask students if they remember the formula $RT=D$, and have them solve for $R$.

Ask the class to create a horizontal line. Why did their approach create or not create a horizontal line? What must be true to have a horizontal line and why? Challenge the students to make a horizontal line higher on the display or lower. What is true about the horizontal lines (they are parallel)? Ask the students to calculate the rate of change of the person who created a horizontal line. Make the connection that the rate of change or slope of a horizontal line is zero.

Ask the class to create a vertical line. Many students will have ideas of how to do this, but ultimately they will not be successful. Let them experiment with this. Challenge the class to explain why this cannot be done. Have a discussion about the fact that it is physically impossible to be in two places simultaneously. Connect this with the idea that a vertical line is not a function. Ask the class what the slope of a vertical line would be. Make the connection that the slope of a vertical line is undefined. (See question 4 in Activity 4.1.3.)

Continue with questions 5–7 in Activity 4.1.3, which direct students to think more precisely about rate of change in terms of meters per second by graphing distance and time on a coordinate plane with precise values labeled on the distance and time axes.
Assess student understanding at the end of the class by displaying a new graph from Data Match and having students write an exit slip describing the walk of the person that would match that graph. Or, you may have students do complete Exit Slip 4.1.2 requiring them to sketch a graph given a verbal description of a person’s walk. Activity 4.1.4 Stories and Graphs may be assigned for homework.

Feel free to modify the activities so that they contain topics of greater interest to your students. Optional ideas include asking students to give a rough description and graph a common destination, such as the walk or car ride from school to a favorite soda shop, or an amusing occurrence, such as a person leaving home heading to school, discovering they left their homework on the kitchen table, returning home, and then proceeding to school at a faster rate.

Differentiated Instruction (For Learners Needing More Help)

Before asking students to describe the motion depicted in a graph, have students brainstorm a list of words to choose from such as: move steadily, speed up, slow down, walk at a constant rate, move away from the motion detector, move toward the motion detector, increasing/decreasing the distance from the motion detector, velocity, steep line, flat line, etc.

To facilitate students trying to translate from one representation of a function to another, you may provide four verbal descriptions, four graphs and four tables, and ask students to match the descriptions, graphs and tables.

Have students annotate a graph with descriptions of the walk or movement rather than write a paragraph describing the walk.

Differentiated Instruction (Enrichment)

Challenge students to show how to walk a lower case letter “m”. Can you make an upper case letter “M” with sharp angles? (No, because you can’t change direction instantly – you always need to slow down turn change direction.) What letters can you walk? Students should notice which letters are functions, which ones aren’t, which have sharp points, and which are rounded.

When students compute velocity by looking at the table for time distance data, a) have them compute the rate of change between each time interval, then have them compute the averages of these rates, and b) have them convert feet per second to miles per hour (or meters per second to kilometers per hour.)

Alternatively, you can distribute the Activity 4.1.5 Motion Graph Challenge Problems.
Group Activity

Data match can be done as a group activity whereby each group discusses their ideas about how to walk to match a given graph, writes down their idea and then tests the idea with the motion detector.

Collecting distance-time data by hand can be done in groups. Students can go into a hallway to time and measure distances they walk. Have them record the data for distance as a function of time in table form, then draw a graph by hand.

If you have access to several motion detectors, you might choose to have students work in small groups to “do the walks”.

Journal Entry

Sketch a motion-time graph of your teacher’s drive to school this morning. Your teacher describes their drive below:

“I started out at 6:30 AM, driving on local streets. Just before I got onto the highway, I stopped at the convenience store to get coffee for $1.05 because I use my eco friendly reusable travel mug (cream no sugar, thank you). After a minute or two on the highway, I realized I forgot my backpack, returned home, retracing my path. With my backpack next to me, I finally get back onto the highway, and was enjoying NPR Morning Edition, when I saw police helping a distressed motorist whose car had broken down in the right lane. All of the traffic was forced to slow down considerably for a short while. Soon enough, I was able to go the normal speed limit of 65mph. The school is only 5 minutes from the highway, so I arrived at school at 7:30, too late for homeroom, but just in time for algebra class.”

Resources and Materials

- Activity 4.1.1 Motion Graphs-What Makes a Function Linear?
- Activity 4.1.2 Motion Graph Scenarios
- Activity 4.1.3 More Motion Graphs
- Activity 4.1.4 Stories and Graphs
- Activity 4.1.5 Motion Graph Challenge Problems
- Exit Slips 4.1.1 and 4.1.2
- Motion detector such as those by Vernier, together with either (a) a calculator with view screen to project to class OR (b) a computer that projects to entire class.
The calculator software is under the APPS key. You may see CBL/CBR or Ranger, for example. You can also download programs like RANGER, HIKER or DATA MATCH from the website [www.Education.TI.com](http://www.Education.TI.com).

The software for the computer called LOGGER LITE is free on the Vernier website [http://www.vernier.com/products/software/logger-lite/](http://www.vernier.com/products/software/logger-lite/). To connect the motion detector to the computer, you will need a printer cable commonly used for many printers. The same printer cable is also sold as the Go Motion cable at Vernier.

- Timer, tape measure and chalk to mark floor location as person walks
- Bulletin Board for key concepts
- Student Journals
- [http://illuminations.nctm.org/LessonDetail.aspx?ID=U188](http://illuminations.nctm.org/LessonDetail.aspx?ID=U188) for an NCTM Illuminations lesson entitled “Movement with Functions”
- Time-Distance lessons under “Classroom Activities” on the Texas Instruments website: [http://education.ti.com/calculators/downloads/US/Activities/Search/Keywords?k=time+distance+graphs](http://education.ti.com/calculators/downloads/US/Activities/Search/Keywords?k=time+distance+graphs)
- Workbooks from Texas Instruments such as activities 1 and 13 in *Real World Math Made Easy* by Chris Brueningsen et. al. or *CBR Explorations: Math and Science in Motion* by Brueningsen et. al. or *CBR Explorations: Modeling Motion: High School Activities with the CBR* by Linda Antinone et. al.
What Makes a Function Linear?

**Linear functions** have graphs that are straight lines while **nonlinear functions** have graphs that are NOT straight lines. If a graph is made up of two or more pieces of lines, then that graph is a special type of linear function called a **piecewise linear function**.

1. Determine which graphs are linear and which graphs are nonlinear.

   ![Graphs A, B, C, D](image)

   **Distance-time functions** describe the distance between a person and an object over time. A distance-time function may be linear or non-linear, increasing, decreasing, or constant, depending on the type of movement. To describe a distance-time function, tell (a) where the object starts, (b) what direction it moves, (c) how fast it moves and (d) whether it is speeding up, slowing down or moving at a steady rate.

2. Suppose a person’s distance from a motion detector is changing over time.
   a. Identify the independent variable in this situation.

   b. Identify the dependent variable in this situation.
We will now create graphs of distance-time functions to match descriptions of movements. The graphs should show a person’s distance from a motion detector sensor over time.

3. Sketch the distance-time graphs for the following scenarios.

   a. Stand one meter from the sensor, walk at a constant (steady) slow pace away from the sensor.

   b. Stand one meter from the sensor, walk away from the sensor changing your pace from slow to fast.

   c. Stand one meter from the sensor, and as you walk away, change your pace from fast to slow.

   d. Stand five meters from the sensor and walk toward the sensor at a constant rate.
e. Stand five meters from the sensor and walk towards the sensor slowly at first, then speed up.

\[ d \]

\[ t \]

f. Stand five meters from the sensor and walk toward the sensor quickly at first, then slow down.

\[ \sim \]

\[ t \]

g. Stand one meter from the sensor and stand still the whole time.

\[ \sim \]

\[ t \]

h. Stand one meter from the sensor and stand still for 3 seconds, then walk away at a constant rate.

\[ \sim \]

\[ t \]

4. a. Describe any similarities among the graphs in 3a, 3b and 3c.

b. What are differences between the graph 3a and the graphs 3b and 3c?

5. Which of the graphs in question (3) could be considered linear and which are nonlinear?

Linear:  

Nonlinear:
6. Describe a scenario of someone walking/running that could create the graphs below.

A

\[
\text{Distance} \quad \text{Time}
\]

7

A:

B

\[
\text{Distance} \quad \text{Time}
\]

2

B:

7. Describe a motion that creates a linear function.

8. Describe a motion that creates a non-linear function.
9. The following table has values collected from measuring a person who attempted to walk at a constant rate. Use the data to determine whether or not the person was successful. Support your answer with a graph.

<table>
<thead>
<tr>
<th>Time (# of seconds)</th>
<th>Distance (# of feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>4.1</td>
</tr>
<tr>
<td>3</td>
<td>5.1</td>
</tr>
<tr>
<td>4</td>
<td>6.2</td>
</tr>
<tr>
<td>5</td>
<td>7.1</td>
</tr>
<tr>
<td>6</td>
<td>8.1</td>
</tr>
</tbody>
</table>

![Graph of time versus distance](image)
Motion Graph Scenarios

1. Sketch the distance-time graphs for the following scenarios:

   a. You start far away from the motion detector and decrease your distance from the motion detector at a steady rate.

   ![Graph](image1.png)

   b. You start near the motion detector and walk away from it, first slowly and then faster.

   ![Graph](image2.png)

   c. You are sitting still 3 meters from the motion detector and don’t move.

   ![Graph](image3.png)

   d. You walk away from the motion detector and then turn around and walk toward it.

   ![Graph](image4.png)

2. Fill in the blanks:

   a. If the motion graph is a straight line, then the walker’s speed must be ______________.

   b. A steep line is caused by a person walking ________________________________.

   c. A decreasing line is caused by a person walking ________________________________.

   d. A horizontal line is caused by a person who ________________________________.

   e. An increasing line is cause by a person walking ________________________________.
More Motion Graphs

In a distance-time graph, the time that has elapsed since the motion began is the independent variable and is on the horizontal axis. The distance from the starting point is the dependent variable and is on the vertical axis.

The rate of change is how far the unit moves compared to how much time it took to move that far:

$$\text{RATE OF CHANGE} = \frac{DISTANCE \text{ travelled}}{\text{Length of TIME it took to travel that distance}}$$

1. How do you have to walk so the motion detector graphs a straight line? Explain as clearly as you can.

2. What determines the steepness of the lines we created as a class using the motion detector?

3. What determines whether the graph is increasing or decreasing?

4. Is it possible to create a vertical line with the motion detector? Explain.
5. Sketch the distance-time graphs for the following situations:

   a. You start 4 m from the motion detector and immediately walk away from it at 1 m/s.
   
   b. You are sitting still about 3 m from the motion detector and don’t move.

6. Describe what’s happening in the following distance-time graphs. Include values!

   a.
   
   b.
7. Describe what’s happening in the following distance-time graphs. Include values!

a. 

\[
\begin{array}{c|c|c|c|c|c}
\text{Time (in seconds)} & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{Distance (in meters)} & 0 & 2 & 4 & 6 & 8 & 10 \\
\end{array}
\]

b. 

\[
\begin{array}{c|c|c|c|c|c}
\text{Time (in seconds)} & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{Distance (in meters)} & 6 & 4 & 4 & 6 & 8 & 10 \\
\end{array}
\]

8. Sketch a distance-time graph of your own design and describe the situation below.

\[
\begin{array}{c|c|c|c|c|c}
\text{Time (in seconds)} & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
\text{Distance (in meters)} & & & & & & \\
\end{array}
\]

Describe the situation.
9. Describe your trip to school this morning starting with your walking out your front door and ending with your arrival at school. (Assume that your path to school is a straight line.)

10. Sketch a distance-time graph based on your description of your trip to school this morning. Label and scale the axes appropriately.
Stories and Graphs

1. On a separate sheet of paper, write a story that describes the following graph. (Assume that your motion takes place along a straight line.)

2. Draw a graph that represents the following story.

You and your father are leaving Valley Plaza on your way home. On the way home you stop at McDonald’s for a bite to eat. After that, you drive three blocks to the gas station but discover that you accidentally left your shopping bag at McDonald’s. So while your father pumps the gas, you walk back to McDonald’s, get your bag, then walk back to the gas station. After your car is filled up you both drive home. (Assume that Valley Plaza, McDonald’s, the Gas Station, and your house are all on the same street).
Distance from house (in feet)

Time since leaving Valley Plaza (in minutes)
**Motion Graph Challenge Problems**

1. Sketch a motion graph of a Cheetah stalking its prey if his hunt was successful. You can watch a video of a cheetah hunting at [http://www.extremescience.com/cheetah.htm](http://www.extremescience.com/cheetah.htm). A cheetah will lie in wait in the grasses of the Serengeti Plain for a gazelle to come near. Within 3 seconds the cheetah will be running at 60 mph as he steadily and rapidly closes in on his prey.

![Diagram](attachment:cheetah_motion_graph.png)

2. Suppose you paint a red spot on a 16” diameter bicycle tire. Sketch a graph showing how far the spot is above the road as a function of time.

![Diagram](attachment:bicycle_tire_motion_graph.png)

3. Suppose you stand 8 meters away from the teacher’s desk. You walk toward the desk steadily and quickly for 3 seconds. Then you stop for 2 seconds to pick up a pencil you dropped. Then you continue to walk slowly toward the desk for the last 5 seconds. What will your motion graph look like? Sketch it below.

![Diagram](attachment:teacher_desk_motion_graph.png)
Unit 4: Investigation 2  (4 Days)

RECOGNIZING LINEAR FUNCTIONS FROM WORDS, TABLES AND GRAPHS

CCSS: F-IF6, F-LE1, F-LE1A

Overview
Students recognize linear functions in tabular and graphical forms and represent functions with verbal descriptions, equations, graphs, and tables. Students will develop methods for identifying the characteristics of linear functions and an understanding of rate of change and initial value in a real word context.

Assessment Activities

Evidence of Success: What Will Students Be Able to Do?
- Distinguish between a linear and non-linear function from a table of values and from a graph.
- Transform a function from one representation to another.
- Identify a linear function’s constant average rate of change and y-intercept and interpret them in a non-contextual setting.
- Use an equation or a graph of a function that models a real world situation to produce a particular ordered pair and give an appropriate interpretation of its meaning in context.
- Choose appropriate increments and scales to construct tables and four-quadrant graphs and select the appropriate table set up and windows when using technology and use the trace feature to demonstrate the relationship between an ordered pair and a point on the graph.

Assessment Strategies: How Will They Show What They Know?
- Exit Slip 4.2 asks students to identify a function as linear or non-linear from a table and to explain their reasoning.
- Journal Entry asks students to compare arithmetic and geometric sequences with linear and non-linear functions.
- Quiz on Investigations 4.1 and 4.2 assesses key ideas from the first two investigations.

Launch Notes
Begin the lesson by showing a variety of pizza menus and leading a short class discussion about what size pizza is the best buy. Now distribute Activity 4.2.1 Pizza Problems. This activity asks students to complete a table of values for the area of a pizza as a function of radius (non-linear), and for the circumference of a pizza as a function of radius (linear). Ask students to specify the domains of the relations and decide if these are functions and if the functions are linear or non-linear.
Closure Notes
Be sure that students can use multiple representations of a function to distinguish linear from non-linear functions. They should be able to transform functions between multiple representations and use each representation to confirm whether or not a function is linear or not. Develop a deep understanding of the idea of constant average rate of change, the y-intercept, increasing and decreasing functions, and the meaning of a coordinate pair in context. Avoid providing students a formula until the ideas makes sense to them. The slope formula is introduced in Investigation 3 and the slope-intercept form of the line is introduced in Investigation 4.

Teaching Strategies

1. Following the launch, display two data tables of your choice, one a linear function and the other a non-linear function. Ask students to consider how they might identify linearity solely from a function’s data table. At first, students will probably note that a function is linear if the x-values and the y-values in a table each form an arithmetic sequence.

   Next, present more data tables including at least one in which the increment between x-values is constant and not one (Δx not necessarily equal to 1). You can use Activity 4.2.2 Recognizing Linear Functions. Allow students time to identify whether or not each table represents a linear relationship and discuss the rationale used.

   Then ask students to describe how they could identify linearity if the data table has x-values with increments that are NOT constant. Allow students time to develop the method for finding the average rate of change between two data points. Ask probing questions so that you guide students to compare various changes in the y-values with the corresponding changes in the x-values, but save the definition and formula for slope for the next investigation. Students might calculate and compare average rates of changes among data tables that represent the special functions examined in Unit 3. This may require a review of equivalent fractions. Students will be able to articulate that data is linear if and only if the rate of change between any two points is the same.

   Students should continue to investigate a variety of linear models (positive slope and positive y-intercept, negative slope and positive y-intercept, positive slope and negative y-intercept, negative slope and negative y-intercept). For each case, students construct a table of ordered pairs given a verbal description of the function, analyze how changes in one variable impact the other variable, plot the corresponding ordered pairs to form a line, and then analyze the line in the context of the real world situation. Students will observe that the graph of a line is increasing when the slope is positive and decreasing when the slope is negative.

   Next, students will examine the relationship between a function’s equation, table and graph. Given equations, students will first construct tables by hand. Negative values of x will be included as inputs. The teacher will then demonstrate how to use the TABLE
feature of the calculator, including the difference between “Ask” and “Auto”, and how to set up the table with the appropriate initial value (TblStart) and increment (ΔTbl). Students will use their calculators to check tables made by hand. Stress that when we compute average rates of change for linear functions (from a table or the graph of the function), the average rate of change never changes regardless of the two points used on the line. Mathematicians refer to this constant value as the slope of the line.

Then, students will use the tables to draw four-quadrant graphs by hand, using the standard window (-10 ≤ X ≤ 10; -10 ≤ Y ≤ 10) introduced in Unit 3. They will then use the calculator to display the same graphs in the same window (Zoom 6). Demonstrate how to adjust the window and ask students to experiment displaying the same function in different windows. Students will describe how the visual representation changes and discuss what makes a “good” window. Students will also learn to use Zoom Decimal and Zoom Integer to create “square” windows. They will notice that in a square window the line \( y = x \) makes a 45 degree angle with the positive x-axis. Students will also explore connections between tables and graphs using the vertical split screen (Mode G-T) and the TRACE feature of the calculator. The teacher may guide students to use windows that give “friendly” outputs, e.g. (-4.7 ≤ X ≤ 4.7; -10 ≤ Y ≤ 10) for \( Y_1 = 2x - 5 \). Students will trace along the graph while reading corresponding values from the table.

For homework you may assign Activity 4.2.3 Using Tables to Determine If a Function Is Linear.

II. Students should continue engaging in activities that highlight the capability of linear functions to model a wide range of real world situations. They can do Activity 4.2.4 Draining a Swimming Pool and Activity 4.2.5 Ordering DVD’s. At least one of these might be assigned for homework. Students will match verbal descriptions, graphs and tables. Students will be able to identify (a) the constant average rate of change, (b) increasing/ decreasing functions, (c) positive and negative y-intercepts, (d) domain and range and (e) how these features of a linear function may be interpreted in the real world context of the given problem.

Note: When identifying x-intercepts often the table of values may not include the x-intercept or the y-intercept despite the fact that one is visible on the graph. Or a table of values might include the x-intercept or y-intercept but the portion of the line graphed may not. Students need to discuss the fact that just because a graph or table does not explicitly show the x- or y-intercept DOES NOT necessarily mean the function does not have one. However, there are some applications where the intercepts do not make sense, for example the relationship between the number of sides of a polygon and the sum of the interior angles in Activity 4.2.6 below.

Exit Slip 4.2 asks students to identify linear functions from tables.
III. In Activity 4.2.6 Linear Functions in Geometry, students explore patterns concerning the relationships of angles in polygons that are modeled with linear functions. The sum of the interior angles of a convex polygon is a function of the number of sides: \( f(x) = 180x - 360 \). To discover this, students can triangulate polygons on a separate worksheet, fill in a table, and make a graph. The fact that non-integral and non-positive values of \( x \) do not make sense in this context should be emphasized. With the screen (0 \( \leq X \leq 9.4; -500 \leq Y \leq 1500 \)) students can use TRACE to find pairs where \( x \) representing the number of sides is an integer in the range 3 \( \leq x \leq 9 \). Students will be asked to make statements relating the graph to context such as, “The point (5, 540) on the graph indicates that if the polygon has 5 sides, then the sum of its interior angles is 540 degrees.” Note: Requisite prior knowledge: the sum of the interior angles of a triangle is 180 degrees.

In a right triangle the measure of one acute angle is a function of the measure of the other acute angle: \( f(x) = 90 - x \). Here the variable \( x \) is continuous with domain: 0 \( < x < 90 \). An appropriate window is (0 \( \leq X \leq 94; 0 \leq Y \leq 100 \)). Students will be asked to make statements such as “The point (42, 48) on the graph indicates that if one of the acute angles measures 42°, then the other one measures 48°.

For more practice, give students Activity 4.2.7 Teddy Bear Sales

At this point students should be ready for Unit 4 Investigations 1 and 2 Quiz. You may want to select only some items from those provided depending upon how much time you want to give for this assessment.

**Differentiated Instruction (For Learners Needing More Help)**
Have students make note cards that they can use as reference while doing the exercises. For example one card will picture the graph of an increasing function with the word “increasing” and a bicycle driving up the hill of the graph. Another card entitled “\( y \)-intercept” can display the \( y \)-intercept on a graph, the coordinate pair \((0,\,?)\) and a table with the \( y \)-intercept circled. A third card entitled “average rate of change” can show a graph, a table and the words \( \frac{\text{changes in } y \text{ values}}{\text{changes in } x \text{ values}} \). Have students identify which card to use as they answer each question in an exercise. Also put these key ideas on a bulletin board.

**Differentiated Instruction (Enrichment)**
Students can begin to invent their own real world situations using events they are involved in. They may invent their own linear scenarios based on fundraisers they are involved in, sports they are playing, or academic content they are studying in their other classes. Challenge students to create an exercise for their classmates to work on, or have them put a power point together analyzing a timely local event that they can show the class.
**Group Activity**
Several of the activities in this investigation, particularly Activities 4.2.3 and 4.2.4 are well suited for group work. Use the jigsaw model of group work and have students complete problems in newly formed groups. Then have the students report back to their original group on the activity that they did or report to the class about their activity.

**Journal Prompt**
Compare the patterns of arithmetic and geometric sequences that you learned in Unit 1 with linear and non-linear functions that you are learning in Unit 4.

**Resources and Materials**

- **Activity 4.2.1** Pizza Problems
- **Activity 4.2.2** Recognizing Linear Functions
- **Activity 4.2.3** Using Tables to Determine if a Function is Linear
- **Activity 4.2.4** Draining a Swimming Pool
- **Activity 4.2.5** Ordering DVD’s
- **Activity 4.2.6** Linear Functions in Geometry
- **Activity 4.2.7** Teddy Bear Sales
- **Exit Slip 4.2** Is it Linear?
- **Unit 4 Investigations 1 and 2 Quiz**
- Straight Edges for drawing linear graphs
- Bulletin Board for key concepts
- Student Journals
- Graphing Calculators
Pizza Problems

Solve the following problems by finding the circumference and area of different sizes of pizzas.

1. Complete the table below using the formula: \( C = 2\pi r \)

<table>
<thead>
<tr>
<th>Radius (inches)</th>
<th>Circumference (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

2. Plot the data from the table above on the coordinate plane below. Label and scale the axes.

3. Is there a linear relationship between the circumference of a pizza and the radius? Explain.
4. Complete the table below using the formula: \( A = \pi r^2 \)

<table>
<thead>
<tr>
<th>Radius (inches)</th>
<th>Area (square inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

5. Plot the data from the table above on the coordinate plane below. Label and scale the axes.

6. Is there a linear relationship between the area of a pizza and the radius? Explain.
7. Is paying $20 for a large pizza, which is 18 inches in diameter, a better buy than paying $5 for a personal size pizza that is 6 inches across? Explain.

8. Suppose someone eats only the stuffed crust and gives the center of the pizza to a friend. How can they get the most crust for their money? Justify your answer.
Recognizing Linear Functions from Words, Tables and Graphs

Linear functions have a constant rate of change. You can determine if a table contains a linear function by calculating rates of change. If the same number is added or subtracted to the dependent variable (the \( y \)-values), whenever the independent variable (the \( x \)-values) changes by some constant amount, the table contains a linear function. The table below contains a linear function because the \( y \)-values decrease by 3 whenever the \( x \)-values increase by 1.

\[
\begin{array}{cccccccc}
 x: & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 y: & 8 & 5 & 2 & -1 & -4 & -7 & -10 \\
\end{array}
\]

\[
rate of change = \frac{change in y}{change in x} = \frac{-3}{1} = -3
\]

1. Which graph shows a linear function? Explain.
2. Plot the points from the given table. State whether the graph represents a linear function.

a. 

\[
\begin{array}{c|ccccc}
 x & 1 & 2 & 3 & 4 & 5 \\
 y & 0 & 2 & 4 & 6 & 8 \\
\end{array}
\]

Linear Function (Yes / No)

Why?

If it is linear what is \( \frac{\text{change in } y}{\text{change in } x} \) between coordinate pairs?

b. 

\[
\begin{array}{c|ccccc}
 x & 1 & 2 & 3 & 4 & 5 \\
 y & 0 & 1 & 3 & 6 & 10 \\
\end{array}
\]

Linear Function (Yes / No)

Why?

If it is linear what is \( \frac{\text{change in } y}{\text{change in } x} \) between coordinate pairs?

c. 

\[
\begin{array}{c|ccccc}
 x & 0 & 1 & 3 & 4 & 5 \\
 y & 9 & 6 & 0 & -3 & -6 \\
\end{array}
\]

Linear Function (Yes / No)

Why?

If it is linear what is \( \frac{\text{change in } y}{\text{change in } x} \) between coordinate pairs?
3. Now that you have seen the graphs of several function, explain how you can determine if a function is linear just by looking at its table.

4. Determine if the table represents a linear function.
   
a. \[
   \begin{array}{cccccc}
   x & 5 & 4 & 3 & 2 & 1 \\
   y & 0 & -1 & -2 & -3 & -4 \\
   \end{array}
   \]
   (Yes / No) Why?

b. \[
   \begin{array}{cccccc}
   x & 0 & 5 & 10 & 20 & 30 \\
   y & 18 & 14 & 10 & 2 & -6 \\
   \end{array}
   \]
   (Yes / No) Why?

c. \[
   \begin{array}{cccccc}
   x & -4 & -2 & 0 & 2 & 4 \\
   y & 3 & 6 & 11 & 18 & 27 \\
   \end{array}
   \]
   (Yes / No) Why?

d. \[
   \begin{array}{cccccc}
   x & -4 & -1 & 0 & 2 & -2 \\
   y & 7 & 1 & -1 & -5 & 3 \\
   \end{array}
   \]
   (Yes / No) Why?
5. Use the equation to complete the table. Tell whether the relationship is linear. If it is linear, identify \( \frac{\text{change in } y}{\text{change in } x} \). If the function is not linear, explain why.

a. \( y = 3x - 8 \)

<table>
<thead>
<tr>
<th></th>
<th>-4</th>
<th>-3</th>
<th>-1</th>
<th>0</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

If it is linear what is \( \frac{\text{change in } y}{\text{change in } x} \) between coordinate pairs?

If not linear, why?

b. \( y = 4(x - 7) + 6 \)

<table>
<thead>
<tr>
<th></th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>13</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

If it is linear what is \( \frac{\text{change in } y}{\text{change in } x} \) between coordinate pairs?

If not linear, why?

c. \( y = x(x - 2) \)

<table>
<thead>
<tr>
<th></th>
<th>11</th>
<th>8</th>
<th>5</th>
<th>2</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

If it is linear what is \( \frac{\text{change in } y}{\text{change in } x} \) between coordinate pairs?

If not linear, why?
6. Every Friday, the mechanics for Griswold Public Schools record the miles driven and the gallons of gas used for each school bus. One week, the mechanics record these data.

<table>
<thead>
<tr>
<th>Gas Used (gallons)</th>
<th>5</th>
<th>8</th>
<th>11</th>
<th>14</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles Driven (miles)</td>
<td>30</td>
<td>48</td>
<td>66</td>
<td>84</td>
<td>102</td>
</tr>
</tbody>
</table>

a. Plot the data from the table on the coordinate plane below. Label and scale axes.

b. Is there a linear relationship between gas used and miles driven? Explain.

c. Use your graph to predict how much gas will be used if the bus is driven 6.5 miles.
Using Tables to Determine If a Function Is Linear

**Linear functions** have a **constant rate of change**. You can determine if a table contains a linear function by calculating rates of change. If the same number is added or subtracted to the dependent variable (the y-values), whenever the independent variable (the x-values) changes by some constant amount, the table contains a linear function. The table below contains a linear function because the y-values decrease by 3 whenever the x-values increase by 1.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>-1</td>
<td>-4</td>
<td>-7</td>
<td>-10</td>
</tr>
</tbody>
</table>

rate of change = \( \frac{\text{change in } y}{\text{change in } x} \) = \( \frac{-3}{1} = -3 \)

1. Determine if each table represents a linear function. Explain why or why not. Comment on the change in y over the change in x between coordinate pairs.

   a.  
   
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-4</td>
<td>-16</td>
<td>-36</td>
<td>-64</td>
</tr>
</tbody>
</table>

   Linear Function (Yes / No)

   Why?

   b.  
   
<table>
<thead>
<tr>
<th>x</th>
<th>-5</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>20</td>
<td>17</td>
<td>14</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

   Linear Function (Yes / No)

   Why?

   c.  
   
<table>
<thead>
<tr>
<th>x</th>
<th>9</th>
<th>6</th>
<th>3</th>
<th>0</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-15</td>
<td>-11</td>
<td>-7</td>
<td>-3</td>
<td>1</td>
</tr>
</tbody>
</table>

   Linear Function (Yes / No)

   Why?
d. 

<table>
<thead>
<tr>
<th></th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y</strong></td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

Why?

2. Use the equation to complete the table. Tell whether the relationship is linear. Explain.

a. \( y = 7 - 4x \)

<table>
<thead>
<tr>
<th></th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

Why?

b. \( y = -\frac{3}{4}x + 2 \)

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

Why?

c. \( y = x(3 + x) \)

<table>
<thead>
<tr>
<th></th>
<th>12</th>
<th>8</th>
<th>4</th>
<th>0</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>x</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Linear Function (Yes / No)

Why?
d. \( y = 3(6x - 1) \)

\[
\begin{array}{cccccc}
 x & 11 & 8 & 5 & 2 & -1 \\
 y & & & & & \\
\end{array}
\]

Linear Function (Yes / No)

Why?

3. For tax purposes, each year Taco Bell keeps track of the total income they bring in and the money they pay to keep the restaurant chain running.

<table>
<thead>
<tr>
<th>YEAR</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income (in millions)</td>
<td>24.3</td>
<td>27.7</td>
<td>31.1</td>
<td>34.5</td>
<td>37.9</td>
</tr>
<tr>
<td>Expenses (in millions)</td>
<td>2.8</td>
<td>5.7</td>
<td>3.4</td>
<td>6</td>
<td>5.2</td>
</tr>
</tbody>
</table>

a. Is there a linear relationship between year and income? Explain.

b. Is there a linear relationship between income and expenses? Explain.

c. Is there a linear relationship between year and expenses? Explain.
**Draining a Swimming Pool**

A backyard pool is 40 feet long, 25 feet wide and 5 feet deep.

1. Find the volume of the pool in cubic feet. To find the volume you must use the formula of a rectangular prism: \( Volume = length \times width \times depth \). Make sure to attach \( ft^3 \) at the end of your answer.

2. There are 7.5 gallons in one cubic foot (1 \( ft^3 \)). Find the volume of the pool in gallons.

The pool is full of water. It is the end of the summer and you need to drain the pool in preparation for winter weather. When the drain is open, **500 gallons flow out every minute**. The amount of gallons of water in the pool is a function of the time (in minutes) it is draining.

3. What is the independent variable?

4. What is the dependent variable?

5. Complete the table. *(Note that time is increasing by 10 minute increments).*

<table>
<thead>
<tr>
<th>Time (in minutes) ( x )</th>
<th>Water in Pool (in gal.) ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>70</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>
6. Use the table to answer the following questions.

a. As the time increases, what happens to the amount of water in the pool; does it increase or decrease?

b. Use the table to find how much the amount of water in the pool changes every 10 minutes.

c. How long will it take the pool to drain completely? Explain your reasoning.

d. In this situation does it make sense to have a negative number in the second column? Explain.

e. Which of the following would be a reasonable domain for this function?

   i. 0 gallons – 37,500 gallons
   ii. 0 minutes – 75 minutes
   iii. \{0, 10, 20, 30, 40, 50, 60, 70, 80\}

f. Which of the following would be a reasonable range for this function?

   i. 0 gallons – 37,500 gallons
   ii. 0 minutes – 75 minutes
   iii. \{-2500, 2500, 7500, 12500, 17500, 22500, 27500, 32500, 37500\}
7. a. Plot the points in the table. Let \( x \) represent time (in minutes) and \( y \) the number of gallons of water remaining in the pool. Make sure to label your axes and choose an appropriate scale.

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
x & & & & & & \\
\hline
y & & & & & & \\
\hline
\end{array}
\]

b. Draw a line through the plotted points. As you move along the line from left to right, does the line go up or down?

c. Where does the line intercept (touch) the \( y \)-axis?

d. What does this point tell you about the problem?

e. Where does the line intercept (touch) the \( x \)-axis?

f. What does this point tell you about the problem?

g. We can interpret any point on the graph. For example, the point \((0, 37,500)\) tells us there was 37,500 gallons of water at the start. What does the point \((30, 22,500)\) tell us?
Ordering DVD’s

If the points on the graph lie on a straight line, the function is a **linear function**.

To see if a function is linear, compare how the $y$-values change with how the $x$-values change. The **average rate of change** between any two points is $\frac{\text{change in } y}{\text{change in } x}$.

If a function is linear, then the $\frac{\text{change in } y}{\text{change in } x}$ between any two points or ordered pairs will always give the same number (the slope) no matter which two ordered pairs you choose.

If a line goes up as we go from left to right on a graph, the line has a positive slope. Notice that as the independent variable $x$ increases, the dependent variable $y$ also increases. We call this an **increasing function**.

If a line goes down as we go from left to right on a graph, the line has a negative slope. Notice that as the independent variable $x$ increases, the dependent variable $y$ decreases. We call this a **decreasing function**.

The point where the line crosses the $y$-axis is called the **$y$-intercept**. This is our “starting point”. Notice that the $y$ intercept is the point always that has an $x$-value of 0.

You are ordering DVD’s from a web site. They charge $14 for each DVD. For any order you must pay an additional $6 for shipping and handling. Your total cost is a function of how many DVD’s you order.

1. What is the independent variable?

2. What is the dependent variable?

3. Fill in the headings and complete the table to the right.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>
4. As the number of DVD’s increases, what happens to the total cost? Does it increase or decrease?

5. Is this function linear? Explain why or why not.

6. If you have only $100 to spend, what is the maximum number of DVD’s you may buy? Explain.

7. a. Plot the points from the data in the table. Let \( x \) represent the number of DVD’s and let \( y \) represent the total cost. Make sure to label your axes and choose an appropriate scale.
b. Draw a line through the plotted points.

c. As you move along the line from left to right, does the line increase or decrease?

d. Where does the line intercept the \( y \)-axis (cost)? What is special about the \( y \)-value at this point?

e. What is the meaning of the point \((5, 76)\)?

f. The equation \( y = 14x + 6 \) can be used to model this situation where \( y \) is your total cost for ordering \( x \) DVD’s. The coefficient of \( x \) is 14. What is the real-world meaning of this number?

g. The constant term is 6. What is the real-world meaning of this number?

h. As the independent variable increases, the dependent variable also increases. Does this mean it is an increasing function or a decreasing function?
Recognizing Linear Functions from Geometric Applications

The sum of the interior angles of a convex polygon is a function of the number of sides in the polygon. The sum of interior angles, \( f(x) \), can be modeled by \( f(x) = 180x - 360 \), where \( x \) is the number of sides in the polygon.

1. Fill in the table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

2. What is a reasonable domain of this situation?

3. What is a reasonable range of this situation?

4. Label your axes. Plot the points (DO NOT CONNECT THE POINTS).

5. Explain why you shouldn’t connect the points.

6. Explain what the number in the shaded box mean in the context of the problem.

7. What is the rate of change of this function?
In a right triangle, the measure of one acute angle is a function of the measure of the other acute angle. The measure of one acute angle, \( f(x) \), can be modeled by \( f(x) = 90 - x \), where \( x \) is the measure of the other acute angle.

8. Fill in the table. Pick your own values for \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. What is a reasonable domain of this situation?

10. What is a reasonable range of this situation?

11. Label your axes. Plot the points.

12. Should you connect the points? Explain why or why not?

13. Explain what the numbers in the shaded box mean in the context of the problem.

14. What is the rate of change of this function?
Teddy Bear Sale

The pep club is planning to raise money by selling stuffed teddy bears for Valentine’s Day. They pay $1500 for a shipment of 300 bears, which they plan to sell for $12 each. They hope to make a profit of at least $1000 but they know that if they don’t sell enough bears they may lose money. They want to explore the relationship between the number of bears they sell and their profit or loss.

For example, if they sell only 100 bears, they take in 100 * $12 = $1200. Their net profit is $1200 minus the $1500 they paid, that is $1200 - $1500 = -$300. This means they would lose $300.

1. What is the independent variable in this situation?

2. What is the dependent variable in this situation?

3. Fill in the headings and complete the following table.

<table>
<thead>
<tr>
<th>Number of Bears Sold</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>-300</td>
</tr>
<tr>
<td>150</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>

4. As the number of bears sold increases, what happens to the profit: does it increase or decrease?

5. As you increase the number of bears sold by 50, does the profit always change by the same amount? What is this amount?
6. Estimate how many bears the club will need to sell to break even (*neither make money nor lose money*).

7. Estimate how many bears the club will need to sell to make their goal of at least $1000.

8. a. Make a graph for the data in the table. Let $x$ represent the number of bears sold and $y$ the profit. Plot the ordered pairs in the table on the graph. Draw a line through the plotted points.

   b. Do all the points appear in the first quadrant? If not, what other quadrant is needed?

   c. As you move along the line from left to right, does the line increase or decrease?
d. Where does the line intercept the $y$-axis? What does this point tell you about the problem?

e. Where does the line intercept the $x$-axis? What does this point tell you about the problem?

f. Use the graph to make a better estimate of how many bears the club will need to sell to make their goal of at least $1000$.

9. The equation $y = 12x - 1500$ can be used to model this situation where $x$ represents the number of bears sold and $y$ represents the profit.

a. The coefficient of $x$ is $12$. What is the real-world meaning of this number?

b. The constant term is $-1500$. What is the real-world meaning of this number?

c. Use the equation to find the number of bears the club will need to sell to make their goal of at least $1000$.

d. Use the equation to find the profit if they sell $180$ bears.
Unit 4: Investigation 3  (4 Days)

CALCULATING AND INTERPRETING SLOPE

CCSS: CCSS: F-IF6, F-LE1a, F-LE1b

Overview
Students discover how to identify the slope of a linear function from a table, two ordered pairs, graph and the verbal description of a linear function. Students also learn how to interpret the slope in the context of real world situations.

Assessment Activities

Evidence of Success: What Will Students Be Able to Do?
- Determine run, rise, and slope given two points in the coordinate plane.
- Identify the slope given the verbal description, graphic or tabular model of a linear function.
- Graph a line given a point and the average rate of change or slope.
- Graph a linear function by creating a table of values when given an equation for the linear function.
- Recognize rates in the form of units of the dependent variable per units of independent variable.
- Interpret the rate of change of the linear function in a real world context.
- Identify and graph horizontal and vertical lines.
- Determine whether lines are parallel or perpendicular.

Assessment Strategies: How Will They Show What They Know?
Exit Slip 4.3.1 assesses students’ understanding of the relationship between slope and rate of change.
Exit Slip 4.3.2 asks students to calculate the slope of a line and use the slope to determine its direction and steepness.
Journal Entry asks students to apply the concept of slope to a previously encountered function in context.

Launch Notes
Break out the motion detector again. Have students do 4 different walks with the motion detector recording their distance as a function of time. One walk should be increasing and steep, the other decreasing and steep. Two other walks should be increasing and almost flat, the other decreasing and almost flat. Using the table feature or the graph’s trace feature, obtain two coordinate pairs on each of the four graphs. Calculate the slope between two points on each of the four walks. Class discussion should draw the connection among fast walks, steeper graphs and slopes of larger magnitude (or absolute value). Slow walks correspond to flatter graphs and slopes smaller in absolute value. Increasing the distance from the motion detector results in a positive slope, whereas decreasing the distance results in negative slope.
Closure Notes
Slope is one of the fundamental concepts in mathematics. By the end of this investigation, students should have a very good understanding of slope and a facility for calculating and interpreting it.

Teaching Strategies

I. In Activity 4.3.1 What is Slope, students transition from a graphical understanding of slope to a numerical formulation of slope. Building upon student understanding of average rate of change and the method developed in Investigation 2 for finding the average rate of change between two data points, students will describe and use the run and rise between two points in the four-quadrant coordinate plane to find the rate of change, or slope. Students will quantify the rise, or the change in $y$, as $y_2 - y_1$ and the run, or the change in $x$, as $x_2 - x_1$, and then calculate the slope of the line between two positions by taking the ratio of these two quantities. The teacher will explain that the letter “$m$” is often used to designate the slope of a line.

You should offer multiple contextual problems and allow students to select specific problems and work individually or in groups. Have students share their results and probe for their understanding of the meaning of the slope in the context of the situations. Students may now calculate slopes from data in the tables and graphs, verify that the coefficient of $x$ is the slope and interpret the slopes as rates of change with appropriate units, e.g., 25 gallons per 10 minutes. Students will learn to express slope in unit rates as well, 2.5 gallons per minute. You may assign the Activity 4.3.2 Calculating and Interpreting Slope.

II. Students continue to explore and practice with the slope formula in Activity 4.3.3 Positive and Negative Slope. They will make the connection between the direction of the graph (increasing, decreasing or horizontal) and the sign of the slope (positive, negative or 0). Students may explore slope as the ratio of rise to run in the context of the pitch of a roof, or the slope of a mountain. Through calculating the slope of a roof using the legs of similar triangles, students will recognize that the slope of a line (or a roof) is the same regardless which two points on the line (roof) are chosen. You may need to discuss the properties of similar triangles in order for students to solve for unknown quantities within the context of the problems. You may assess students’ understanding of the connection between slope and rate of change with Exit Slip 4.3.1.

III. The focus of Activity 4.3.4 Magnitude of Slope is on identifying the steepness of a linear function by the magnitude of its slope. Use a class discussion to develop a rule of thumb for what is considered a steep slope and what is not so steep. Draw lines with slope $\pm 4$ or $\pm 5$ on the board. Then draw lines with slope $\pm \frac{1}{3}$ or $\pm 0.1$. Ask students to vote on which lines are steep and which are not. Sketch more lines with slopes gradually closer to $\pm 1$, continuing to ask students to identify what is steep and what is not so steep. Direct the discussion so that students conclude that the larger the magnitude (the absolute value) of the number associated with the slope
of a line, the steeper the line. You may use Exit Slip 4.3.2 to determine whether students have grasped this relationship.

IV. Next, students should explore tables, graphs and equations of horizontal lines with equations of the form \( y = b \) and vertical equations of the form \( x = a \). The calculator can be used here. Equations for horizontal lines may be entered into the Y= menu; however, for vertical lines the DRAW menu must be used, perhaps only by the teacher as a demonstration. You may begin by presenting the table form of vertical and horizontal lines, and have students review what they have found in Investigation 1 about the slope of each type of line.

Horizontal lines have a slope of zero (\( y = 0x + b \)) (all run, no rise). These give rise to constant functions of the form \( f(x) = b \). You may help students understand the concept of a constant function with these examples. (1) In a snack machine the input is the combination of buttons you push and the output is the item selected. A snack machine filled with identical potato chips may be considered a “constant function.” (2) We have seen that many numerical functions are associated with a rule such as “multiply by 3.” Suppose the rule is “multiply by 0.” Then we get the same output for every input.

Review the idea that vertical lines have undefined slopes (all rise, no run-division by zero is undefined) and horizontal lines have a slope of zero. Note that \( x = a \) is not a function. Though not functions, vertical lines play an important role in algebra. Use Activity 4.3.5 Horizontal and Vertical Lines to develop the concepts. Activity 4.3.6 Additional Practice with Horizontal and Vertical Lines may be assigned for homework.

Differentiated Instruction (For Learners Needing More Help)

Continue putting the major concepts on the index cards and the bulletin board. These include rise, run, positive slope, negative slope, zero slope, undefined slope, steeper, flatter, horizontal, and vertical. Have students choose or point to the concept they need to use when doing an exercise.
Differentiated Instruction (Enrichment)

Students might research the building codes and specifications for the slope of handicap ramps or the slope of a stairway. They can interview a builder and find whether builders refer to slopes of roofs or stairs as rise/run or run/rise. As another extension students might want to explore the construction of Egyptian or Mayan pyramids and compare the incredible geometry and specifications of each structure.

For another extension, students might research the history of “m” used to designate slope of a line. Do other countries use “m” for slope? Have them do a scavenger hunt for slope in math writings in other countries to learn about how universal the language of math is.

Group Activity

For variety, give one exit slip to each group, and have each group member solve one step of the problem, then hand the problem to the next group member. Continue to pass the problem around to each person until the problem is done. One of the rules is no talking, and no interfering with a person as they do their step.

Journal Entry

Choose one of the students’ favorite examples from Unit 2 such as Cab Fares, Weight Loss, Debit Cards, etc., which they analyzed before they knew about the vocabulary and formulas for linear functions. For example, you can ask them to identify the average rate of change, slope, and y-intercept, and state what the slope and y-intercept mean in context, and whether the function is increasing or decreasing.

Resources and Materials

- Activity 4.3.1 What is Slope
- Activity 4.3.2 Calculating and Interpreting Slope
- Activity 4.3.3 Positive and Negative Slope
- Activity 4.3.4 Magnitude of Slope
- Activity 4.3.5 Horizontal and Vertical Lines
- Activity 4.3.6 Additional Practice with Horizontal and Vertical Lines
- Exit Slips 4.3.1 and 4.3.2
- Straight Edges for drawing linear graphs
- Bulletin Board for key concepts
- Student Journals
- Graphing Calculators
What is Slope?

What is slope? If you have ever walked up or down a hill, then you have already experienced a real life example of slope. Keeping this fact in mind, by definition, the slope is the measure of the steepness of a line. In math, slope is defined from left to right.

There are four types of slope you can encounter. A slope can be positive, negative, zero, or undefined.

**Positive slope:**
If you go from left to right and you go up, the line has a positive slope.

**Negative slope:**
If you go from left to right and you go down, the line has a negative slope.

**Zero slope:**
If you go from left to right and you don’t go up or down, the line has a zero slope.

**Undefined slope:**
If you can only go up or you can only go down, the line has an undefined slope.

Here is one method of finding the slope of a line. Remember, slope is a measure of how steep a line is. That steepness can be measured with the following formula.

\[
\text{slope} = \frac{\text{rise}}{\text{run}}
\]

Let’s illustrate with two examples:

For this situation, we see that the rise is 2 and the run is 4.
So, the slope = \( \frac{2}{4} \) or \( \frac{1}{2} \) after simplification. Since \( \frac{1}{2} \) is positive, you are going uphill. Every time you go up 1 unit, you go across or horizontally to the right 2 units.

For this situation, we see that the rise is -5 and the run is 3.
So, the slope = \( -\frac{5}{3} \). Since \( -\frac{5}{3} \) is negative, you are going downhill. Every time you go down 5 units, you go horizontally to the right 3 units.
1. Find the rise and the run for each solid line. Then state the slope of the solid line. Remember, slope is defined from left to right.

   a.  
   
   b.  

   Rise = 
   Run = 
   Slope = 

   Rise = 
   Run = 
   Slope = 

2. Starting at point $A$ find the rise and run to get to point $B$. Then connect the points to make a solid line. Identify the rise, run, and slope for the line segment between each pair of points below.

   a.  
   
   b.  

   Rise = 
   Run = 
   Slope = 

   Rise = 
   Run = 
   Slope = 
3. Use the coordinate plane below.

a. Connect the points using a straightedge. Extend the line past points $A$ and $C$ and place arrows at each end.

b. Find the slope between points $A$ and $B$.

\[
\text{Rise} = \underline{\quad} \quad \text{Run} = \underline{\quad} \quad \text{Slope} = \underline{\quad}
\]

c. Find the slope between points $B$ and $C$.

\[
\text{Rise} = \underline{\quad} \quad \text{Run} = \underline{\quad} \quad \text{Slope} = \underline{\quad}
\]

d. Find the slope between points $A$ and $C$.

\[
\text{Rise} = \underline{\quad} \quad \text{Run} = \underline{\quad} \quad \text{Slope} = \underline{\quad}
\]

e. What can you conclude about the slope of this line looking at your results in parts b thru d?

f. Starting at point $C$ find a fourth point which would belong to the same line. Label your fourth point $D$ and explain how you arrived at it using what you know about slope.
4. Now, let’s see how to find the slope when we don’t know the rise and the run. If we graph the slope on the coordinate system, we will be able to derive another formula for slope using the x and y values of the coordinates.

a. Let’s put a line with a slope of \( \frac{1}{2} \) on the coordinate system.

- Begin by plotting the point (1, 3) and labeling it point A.
- From point A do the rise and run for the slope that is 1/2. Plot this second point, and label it point B.
- Connect the points using a straight edge and name the coordinates of point A and point B.
- Extend the line past points A and B and place arrows at each end.

b. Write the ordered pair for the points: A \((____,____)\) B \((____,____)\)

c. The two coordinates for points A and B can be used to get the slope of \( \frac{1}{2} \).

Let us find the difference in the y-coordinates:

Since we cannot call both coordinates \( y \), we can call one \( y_1 \) and call the other \( y_2 \).

Let \( y_1 \) represent the y-coordinate of point A. Therefore, \( y_1 = \)________  
Let \( y_2 \) represent the y-coordinate of point B. Therefore, \( y_2 = \)________

Now subtract \( y_2 - y_1 = \)________

The difference in the y-coordinates can be expressed as \( y_2 - y_1 \). This is the \textbf{RISE}.

Let us find the difference in the x-coordinates:

Since we cannot call both coordinates \( x \), we can call one \( x_1 \) and call the other \( x_2 \).

Let \( x_1 \) represent the x-coordinate of point A. Therefore, \( x_1 = \)________  
Let \( x_2 \) represent the x-coordinate of point B. Therefore, \( x_2 = \)________

Now subtract \( x_2 - x_1 = \)________

The difference in the x-coordinates can be expressed as \( x_2 - x_1 \). This is the \textbf{RUN}.
The formula for the slope between the two points A and B can be found by using the $x$ and $y$ coordinates of the two points. Call the ordered pair for point A $(x_1, y_1)$ and the ordered pair for point B $(x_2, y_2)$.

$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

5. Use the formula above to find the slope of the line passing through the given points. Show your work.

   a. $(1, 5) & (2, 9)$
   
   $$y_1 = _____ \quad y_2 = _____$$
   $$x_1 = _____ \quad x_2 = _____$$

   b. $(2, 4) & (1, 1)$
   
   $$y_1 = _____ \quad y_2 = _____$$
   $$x_1 = _____ \quad x_2 = _____$$

   c. $(4, 0) & (8, -2)$
   
   $$y_1 = _____ \quad y_2 = _____$$
   $$x_1 = _____ \quad x_2 = _____$$

   d. $(-8, 6) & (3, 4)$
   
   $$y_1 = _____ \quad y_2 = _____$$
   $$x_1 = _____ \quad x_2 = _____$$
Slope is a measure of **steepness** and **direction**. Slope describes a **rate of change**.

6. Todd had 5 gallons of gasoline in his motorbike. After driving 100 miles, he had 3 gallons of gasoline left. The graph below shows Todd’s situation.

![Graph showing gasoline vs miles driven](image)

a. What are the coordinates of two points that you could use to find the slope of the line?

   $A (____,____), \ B (____,____)$

b. What is the slope of the line? Write in fraction form and use the units of measure you find on the $y$ and $x$ axes.

c. Write the slope as a **unit rate** that will be in gallons per mile.
A rate is a ratio that compares two units of measure.

An example of a rate in fraction form is \( \frac{170 \text{ dollars}}{20 \text{ hours}} \). Slopes are rates.
You can rename rates like you rename fractions. In this example divide the numerator and
denominator by 10, to obtain an equivalent rate of \( \frac{17 \text{ dollars}}{2 \text{ hours}} \).
Divide the numerator and denominator by 2 to obtain \( \frac{12 \text{ dollars}}{1 \text{ hour}} \). This is a unit rate, because 1 is
in the denominator.
Writing the fraction in decimal form gives \( \frac{8.5 \text{ dollars}}{1 \text{ hour}} \). In every day language, we say “$8.50 per
hour is the rate of pay.” This is also a unit rate.

One way to obtain a unit rate is to rewrite the fraction so the denominator is 1. You can also
think of renaming the fraction to decimal form.

7. Sam and Kim went on a hike. The graph
at the right shows their situation.
   a. Find the slope of Kim’s hike.
      (Always include units of measure.)
   b. Write Kim’s slope as a unit rate.
   c. Find the slope of Sam’s hike.
   d. Write Sam’s slope as a unit rate.
   e. Who is hiking at a faster speed, Kim or Sam? Explain how you know by looking at the
graph and by using the numbers for slope that you obtained above.
Calculating & Interpreting Slope

The formula for the slope between the two points A and B can be found by using the x and y coordinates of the two points. Call the ordered pair for point A \((x_1,y_1)\) and the ordered pair for point B \((x_2,y_2)\).

\[
slope = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

The slope of a line can always be represented as a fraction. For example, if the slope is 5, we can write 5 as the fraction \(\frac{5}{1}\). That means the rise is “up 5” and the run is “right 1”.

1. Plot the given point on the coordinate plane, and then use the slope to find a second point on the line. Connect the points with a straight line.

   a. Point (2,0), slope = \(\frac{3}{2}\)
   b. Point (-8,4), slope = \(-\frac{1}{4}\)

   ![Graph of point (2,0) with slope \(\frac{3}{2}\)]
   ![Graph of point (-8,4) with slope \(-\frac{1}{4}\)]

   Second point: ___________  Second point: ___________
2. Plot the given point on the coordinate plane, and then use the slope to find a second point on the line. Connect the points with a straight line.

   a. Point (-3, -5), slope = 2
   b. Point (-2, 7), slope = -3

   

   Second point:___________

   Second point:___________

3. Use the slope formula to find the slope of the line passing through the given points. Show your work.

   a. (1, 5) & (2, 9)
   b. (2, 4) & (1, 1)
   c. (0, 4) & (-2, 8)
   d. (8, -8) & (6, 4)
e. (3, -2) & (-7, -2)  
f. (7, -6) & (2, -3)

g. (-3, -2) & (-1, -7)  
h. (2, -6) & (5, -1)

Sometimes it is useful to express slope as a **unit rate**. A unit rate has a denominator of 1. An example of a unit rate is $8 per hour: \frac{8 \text{ dollars}}{1 \text{ hour}} that is the same as \frac{16 \text{ dollars}}{2 \text{ hours}}. In a distance-time function, the slope may be \frac{3 \text{ meters}}{2 \text{ seconds}}, which is also \frac{3}{2} meters per second. As a decimal, it is 1.5 meters per second or \frac{1.5 \text{ meters}}{1 \text{ second}}.

4. Mr. Peel started 7 meters away from the motion detector, and 2 seconds later, he was 3 meters away from the motion detector. The graph below displays Mr. Peel’s motion. Find the **average rate of change** or **slope** in meters per second.
5. Sara started out with $50 in her piggy bank. Every week she deposited the same amount of money in the bank. After 7 weeks she had $67.50. Find the average rate of change or slope in dollars per week. (This is a unit rate.)

6. At 4 a.m. the temperature was 38 degrees F. By 11 a.m. the temperature had risen to 60 degrees F. Find the average rate of change or slope in degrees per hour.
Positive and Negative Slope

The slope between two points on the graph of a function can be found by determining the ratio of the vertical distance and the horizontal distance between the two points. This ratio is known as the \( \frac{\text{change in } y}{\text{change in } x} \) or \( \frac{\text{rise}}{\text{run}} \).

The formula for the slope between the two points A and B can be found by using the x and y coordinates of the two points. Call the ordered pair for point A \((x_1, y_1)\) and the ordered pair for point B \((x_2, y_2)\).

\[
slope = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Label the coordinates of two points on each line. Find the slope of each line by (a) finding the rise and run from the graph, and (b) using the slope formula. State whether the function is increasing or decreasing.

1. 
   
   \[\begin{array}{c}
   \text{By graph:} \\
   (a) \hspace{1cm} \text{slope} = \frac{\text{rise}}{\text{run}} \\
   \text{Rise} = \underline{\hspace{2cm}} \\
   \text{Run} = \underline{\hspace{2cm}} \\
   \text{Slope} = \underline{\hspace{2cm}}
   \end{array}\]

   \[\begin{array}{c}
   \text{By slope formula:} \\
   (b) \hspace{1cm} \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \\
   A (\underline{\hspace{2cm}}, \underline{\hspace{2cm}}) \hspace{1cm} B (\underline{\hspace{2cm}}, \underline{\hspace{2cm}}) \\
   x_1 = \underline{\hspace{2cm}} \hspace{1cm} y_1 = \underline{\hspace{2cm}} \\
   x_2 = \underline{\hspace{2cm}} \hspace{1cm} y_2 = \underline{\hspace{2cm}} \\
   \text{Slope} = \underline{\hspace{2cm}}
   \end{array}\]

   Increasing or decreasing?

2. 
   
   \[\begin{array}{c}
   \text{By graph:} \\
   (a) \hspace{1cm} \text{slope} = \frac{\text{rise}}{\text{run}} \\
   \text{Rise} = \underline{\hspace{2cm}} \\
   \text{Run} = \underline{\hspace{2cm}} \\
   \text{Slope} = \underline{\hspace{2cm}}
   \end{array}\]

   \[\begin{array}{c}
   \text{By slope formula:} \\
   (b) \hspace{1cm} \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \\
   A (\underline{\hspace{2cm}}, \underline{\hspace{2cm}}) \hspace{1cm} B (\underline{\hspace{2cm}}, \underline{\hspace{2cm}}) \\
   x_1 = \underline{\hspace{2cm}} \hspace{1cm} y_1 = \underline{\hspace{2cm}} \\
   x_2 = \underline{\hspace{2cm}} \hspace{1cm} y_2 = \underline{\hspace{2cm}} \\
   \text{Slope} = \underline{\hspace{2cm}}
   \end{array}\]
Increasing or decreasing?

3.

By graph:
(a) $slope = \frac{rise}{run}$

Rise = ________

Run = ________

Slope = ________

By slope formula:
(b) $slope = \frac{y_2 - y_1}{x_2 - x_1}$

A $( , )$ B $( , )$

$x_1 = ________ y_1 = ________$

$x_2 = ________ y_2 = ________$

Slope =

Increasing or decreasing?

4.

By graph:
(a) $slope = \frac{rise}{run}$

Rise = ________

Run = ________

Slope = ________

By slope formula:
(b) $slope = \frac{y_2 - y_1}{x_2 - x_1}$

A $( , )$ B $( , )$

$x_1 = ________ y_1 = ________$

$x_2 = ________ y_2 = ________$

Slope =

Increasing or decreasing?

5.

By graph:
(a) $slope = \frac{rise}{run}$

Rise = ________

Run = ________

Slope = ________

By slope formula:
(b) $slope = \frac{y_2 - y_1}{x_2 - x_1}$

A $( , )$ B $( , )$

$x_1 = ________ y_1 = ________$

$x_2 = ________ y_2 = ________$

Slope =
Increasing or decreasing?
6. Based on your work on the 5 functions above, complete the sentences below with the following words:
   negative  zero  positive
   a. The slope of a horizontal line is ______________.
   b. The slope of an increasing function is ______________.
   c. The slope of a decreasing function is ______________.

7. Scott measured the slope of three roofs in Griswold. To measure the slope of a roof, Scott measured the height of the roof at the corner of the building and measured the height of the roof 12 feet from the corner of the building. Here are the measurements for all three roofs.

<table>
<thead>
<tr>
<th></th>
<th>Stephanie’s House</th>
<th>Thad’s House</th>
<th>Cody’s Horse Barn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from Corner</td>
<td>Roof Height</td>
<td>Distance from Corner</td>
<td>Roof Height</td>
</tr>
<tr>
<td>0</td>
<td>9</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>15</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

a. Plot the points for each person’s roof and connect the points to draw the roof line. Note: Make the scales of each graph the same.
b. Determine the slope (the average rate of change) of each roof. Use units of measures.

Stephanie’s: ________ Thad’s: ________ Cody’s: ________

c. Which building is most likely to have snow pile up on it? Explain.
Magnitude of Slope

In this activity you will identify when a line is steeper than another line by looking comparing equations or comparing sets of points.

1. Solid or Dashed?
   a. Which line do you think is steeper?

   b. Find the slope of each line.

   Slope of solid line:

   Slope of dashed line:

2. Solid or Dashed?
   a. Which line do you think is steeper?

   b. Find the slope of each line.

   Slope of solid line:

   Slope of dashed line:
3. Using the slope formula, calculate the slope of each line.

   (a) (2, 5) and (-2, 10)  
   (b) (3, 1) and (-3, -10)

Which line do you think is steeper, line (a) or line (b)?

4. When selecting between two lines, how can you tell from the slope which line is steeper?

5. When selecting between two lines, how can you tell from the slope which line is flatter?
6. Graph \( y = \frac{2}{3}x + 4 \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

7. Graph \( y = -\frac{5}{2}x + 7 \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

8. Which of the lines in Exercises 6 and 7 is steeper? Explain why.
9. Calculate the slope between the following points using the slope formula \[ m = \frac{y_2 - y_1}{x_2 - x_1} \].

Then, in a few words, tell what the line that passes through the points looks like. Is it steep or not so steep? Is it increasing, decreasing, or horizontal?

   a. (1, 2) and (-2, 11).
   b. (0, 5) and (2, 10)
   c. (1, 1) and (1, -5)
   d. (-8, 1) and (5, 1)

10. Find the slope of each line of the graph using the grid to count the change in \( y \) and the change in \( x \). Then tell if the line is steep or not so steep.

   A:
   B:
   C:
   D:
   E:
Horizontal and Vertical Lines

Warm Up: Simplify:

\[
\begin{align*}
\text{a. } & \quad \frac{3 - 3}{6} = \\
\text{b. } & \quad \frac{3}{6 - 6} = \\
\text{c. } & \quad \frac{3 - 3}{6 - 6} = \\
\end{align*}
\]

State a conclusion:

- If zero is in the numerator of a fraction, but not in the denominator, the fraction equals ____.
- If zero is in the denominator of a fraction, the fraction is ______

1. Tell whether or not the graphs below display a function. Calculate the slope \((m)\) of each line. You may either find the rise and run directly from the graphs or use the slope formula to get your answers. Write your answers as a fraction and then simplify the fraction if possible. 

**Hint:** Pick 2 easy points from each line to work with.

![Graph A](image1.png)  
![Graph B](image2.png)

Function: yes or no?  
Function: yes or no?

Slope of Line A  
Slope of Line B

State a conclusion: The slope of a horizontal line equals ______.
2. Complete a table for each function below and then plot the points from the table on the following coordinate plane. Using a ruler, connect the points on each coordinate plane.

a. \( y = 5 \) is the same as \( y = 0x + 5 \)  

\[
\begin{array}{c|c|c|c|c}
 x & -2 & -1 & 0 & 1 \\
 y & & & & \\
\end{array}
\]

b. \( y = -4 \) is the same as \( y = 0x - 4 \)  

\[
\begin{array}{c|c|c|c|c}
 x & -5 & -2 & 0 & 2 & 5 \\
 y & & & & & \\
\end{array}
\]

\[\text{State a conclusion: An equation of the form } y = \underline{\phantom{00}} \text{ will be a horizontal line.}\]

3. Which of the following equations will give a graph that is a horizontal line? (Circle all that apply.)

a. \( y = \frac{-2}{3}x + 1 \)  
b. \( y = 17 \)  
c. \( y = x \)  
d. \( y = 0x - 1 \)  
e. \( y = 0 \)  
f. \( x = 5 \)

4. The slope formula is:

5. Find the slope between the two points using the slope formula.

a. \((1, -3) \text{ and } (-5, -3)\)  
b. \((-4, 4) \text{ and } (5, 4)\)
6. Without using the slope formula, how can you tell if the slope of a line between two points will be zero just by looking at the two points?

7. Tell whether or not the graph displays a function. Calculate the slope ($m$) of each line. You may either find the rise and run directly from the graph or use the slope formula to get your answers. Write your answer as a fraction and then put the fraction in simplest form. 

**Hint:** Pick 2 easy points from each line to work with.

**Function:** yes or no?  
**Slope of Line A**

**Function:** yes or no?  
**Slope of Line B**

*State a conclusion:* The slope of a **vertical line** is ____________________.
8. Complete a table for each equation below and plot the points from the table on the following coordinate plane. Using a ruler, connect the points on each coordinate plane.

   a. \( x = 5 \) is the same as \( x + 0y = 5 \)  
   b. \( x = -4 \) is the same as \( x + 0y = -4 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

State a conclusion: An equation of the form \( x = \_ \_ \_ \_ \) will be a **vertical line**.

9. Which of the following equations will give a graph that is a vertical line?  
   (Circle all that apply.)

   a. \( x = 50 \)  
   b. \( y = 7 \)  
   c. \( y = 3x + 50 \)  
   d. \( x = 1 \)  
   e. \( x = 0 \)  
   f. \( x + y = 50 \)

10. Find the slope between the two points using the slope formula.

   a. \((1,3)\) and \((1,5)\)  
   b. \((-4,4)\) and \((-4,7)\)

11. Without using the slope formula, how can you tell if the slope of a line between two points will be undefined just by looking at the two points?
Additional Practice with Horizontal and Vertical Lines

The graph of the line $y = b$ is a horizontal line. The slope is 0. The graph of the line $x = a$ is a vertical line. The slope is undefined.

1. Sketch the graphs of the following:
   
   a. $y = 7$
   
   ![Graph of y = 7]

   b. $x = -6$
   
   ![Graph of x = -6]

   What is the slope?  
   Is this a vertical or horizontal line?

   What is the slope?  
   Is this a vertical or horizontal line?

2. Tell which of the following equations will have graphs of horizontal lines, vertical lines or neither. (You can write H, V or N next to each part.)
   
   a. $x = 12$ _____
   
   b. $y = x + 12$ _____
   
   c. $y = 12$ _____

   d. A line with slope zero _____
   
   e. A line with undefined slope _____

   f. A line with slope one _____
   
   g. A line through the points (0,3) and (2,3) _____

   h. A line through the points (-2, 4) and (-2, 5) _____
3. Use the slope formula to find the slope between the two points. Graph the points. Connect them with a line. Find an equation of a line through the points.

a. (3,6) and (2,6)  

b. (6,3) and (6,2)  

Slope = 

Equation of the line:  

Equation of the line:  

c. (-5, -4) and (-5, 7)  

d. (-9, 0) and (8, 0)  

Slope =  

Equation of the line:  

Equation of the line:
Unit 4: Investigation 4 (4 Days)

EFFECTS OF CHANGING PARAMETERS OF AN EQUATION IN SLOPE-INTERCEPT FORM: \(y = mx + b\)

CCSS: F-LE2, F-LE5, F-IF7, F-IF7a, G-GPE 5

Overview
Students examine how changes in the parameters \(m\) and \(b\) in slope-intercept form affect the function’s graph.

Assessment Activities

Evidence of Success: What Will Students Be Able to Do?
- Describe the changes in a line that occur when the \(y\)-intercept increases or decreases.
- Describe the changes in a line that occur when the slope increases or decreases.
- Graph a line given the slope-intercept form of a line by first plotting the \(y\)-intercept then using slope to find a second point on the line.
- Explain the meaning of a change in slope or a change in \(y\)-intercept in the context of a real world problem.
- Identify the slope and \(y\)-intercept of the line from the graph of a linear function.
- Find the slope-intercept form of the equation of a line given its graph with the \(y\)-intercept and an indicated point.
- Identify parallel lines as having the same slope, but distinct \(y\)-intercepts.
- Identify perpendicular lines as having slopes that are opposite reciprocals, or equivalently, slopes whose product is -1.

Assessment Strategies: How Will They Show What They Know?
Exit Slip 4.4.1 assesses student understanding of the parameters \(m\) and \(b\).
Exit Slip 4.4.2 has students apply the slope-intercept form of the line to a real world context.
Journal Entry asks students to think about the conditions that determine a line.

Launch Notes
This may be the first intensive use of the graphing feature of the calculator, so you may want to have a brief discussion about the calculator as a laboratory tool that will help us discover mathematics. It is important to know when it is appropriate, even necessary, to use the calculator and when the calculator may lead you astray. A brief history of computing tools and computing machines in mathematics may be in order. You could have students look up one fact about technology in mathematics and bring that fact to class to share with their classmates. You could assign the following topics to different students and ask them to do a quick search:
Pascal’s calculating machine, the slide ruler, the first computers of various types (main frame, desktop), the first calculators of various types (adding machines, scientific, graphing), costs and capabilities of calculators since the 1960’s, computer algebra systems. Inform students how
advances in technology have changed mathematics and changed which mathematical disciplines are important.

**Closure Notes**
Point out to students that since Unit 1, they have been working with explicit rules for arithmetic sequences and real life scenarios such as the Hydrocarbon activity that involve rates of change ($m$) and fixed starting amounts ($b$). We have moved from an intuitive understanding of the slope-intercept form of a linear function to a more formal mathematical view of $y = mx + b$ which includes mathematical tools and skills students will need to work effectively to understand this ubiquitous pattern.

**Teaching Strategies**

I. Students start **Activity 4.4.1 Effects of Changing Parameters** by graphing a set of linear functions having the same $y$-intercept but different slopes. Students will discover that changing the magnitude of the parameter $m$, the slope of the line, causes changes in the steepness of the graph, and changing the sign of $m$ changes the direction of the graph.

Students will then graph a set of linear functions having the same slopes but different $y$-intercepts on the same set of axes. Students will discover that changing the $y$-intercept of a linear function causes vertical shifts in the function’s graph. Hence $b$ plays two roles – that of the vertical shift and the $y$-intercept. In this activity students may discover that parallel lines have the same slope. This will be discussed later in this investigation.

Help students to arrive at a definition for “parameter,” and help them distinguish parameters from variables. In the case of linear functions, the parameters are $m$ and $b$, and the variables are $x$ and $y$. For any particular linear function the values of $m$ and $b$ are constant. Often they are given or there is enough information to enable us to find them. We can understand the effect of these parameters by letting them vary and comparing one linear function with another. The concept of parameter will be revisited in Unit 7 with exponential functions and in Unit 8 with quadratic functions.

Students should practice drawing accurate graphs and labeling the graphs clearly. Graphing calculators or graphing software can provide a dynamic picture of the changes in the graph as the parameters increase or decrease.

**Exit Slip 4.4.1** assesses what students have learned so far about the parameters $m$ and $b$. It is designed for a group activity in which each pair of students is given a different linear function, but it can be modified for use as an individual assessment.

II. In **Activity 4.4.2 Slope-Intercept Form**, students identify the slope as $m$ and the $y$-intercept as $b$ from multiple representations of linear functions. From the
algebraic equation, have students create a table of values and graph the function by plotting points. From the table or the graph, have them identify the slope and the \( y \)-intercept as they did in Investigation 3. Lead students to discover that the slope is the coefficient of \( x \) and the \( y \)-intercept is the constant term. Encourage students to graph a function in slope-intercept form by plotting the \( y \)-intercept and then using slope to find a second point on the line. Reverse the process by presenting a graph and asking students to identify the slope and \( y \)-intercept and then write the equation. Activity 4.4.3 Practice with Slope-Intercept Form provides additional practice and is suitable for homework. Students should realize that the slope and the \( y \)-intercept are sufficient characteristics for determining a linear function.

In Activity 4.4.4 Making a Profit, students explore a real world application and analyze the effect changes in the slope and the \( y \)-intercept have on a linear function in context.

**Group Activity**

Several of the activities in this investigation, particularly Activity 4.4.4, are well suited for group work. Consider doing a jigsaw puzzle style of group work whereby students from an original group are reorganized into ability groups unbeknownst to the students. Assign a problem to each group according to their ability. Then the original groups reconvene for each student to share their work.

III. Provide additional practice with problems concerning distance versus time or dollars per item so students may interpret the \( y \)-intercept as the “start” and the slope as how things change. Knowing where to start and how to move is sufficient for finding a solution. In previous activities, students used the slope-intercept form of the line without calling it by name. Activity 4.4.5 Applications of Slope-Intercept Form contains questions which students may find familiar as well as questions related to slope-intercept form and interpreting the slope and \( y \)-intercept. This activity could be used as a jigsaw group work activity or may be assigned for homework.

**Exit Slip 4.4.2** prompts students to solve a contextual problem with an equation in slope-intercept form.

IV. To complete this investigation of slope-intercept form, students should discover properties of slopes of parallel and perpendicular lines. To place the idea of parallel lines in context, you might describe two walkers traveling at the same rate of speed but starting from different places. Ask the students if the two would ever meet. Have students graph several pairs of equations in slope-intercept form that are either parallel or perpendicular. You may give them tables of data that will result in parallel and perpendicular lines and have students plot the points and compare the equations to the graphs. See if students can then use the comparisons to explain how to algebraically determine whether lines are parallel or
perpendicular given the equation or the slope of each line. For additional
reinforcement, you may give students, working with a partner, a pair of points on
each of two lines and observe how they decide to determine whether the lines are
parallel or perpendicular. Have students share their results.

**Activity 4.4.6 Parallel and Perpendicular Lines** allows students to explore
relationships between parallel and perpendicular lines and contains questions
similar to the ones outlined above.

If you have students graph perpendicular lines on the calculator caution them that
the lines will look perpendicular only if they use a square grid. Any window on
the calculator may be made square by using the Zoom Square command (Zoom 5
ZSquare) in the Zoom menu.

Additional practice with parallel and perpendicular lines may be found in **Activity
4.4.7 More Parallel and Perpendicular Lines**.

Here are two additional activities which reinforce the main ideas from this
investigation:

1. Students may complete the *Movement with Functions: Lesson 2, How Did I
   Move* activity sheet from *NCTM Illuminations* (or an equivalent activity). This
   kinetic activity requires groups of three students to go to a football field (or
   indoor space containing position markers) to implement specific movement
   scenarios. Each group of students has a stop watch and a set of index cards
   that specifies certain movement tasks. As one student performs a movement
task, another student tracks their position, and a third student records the time.
   Following the activity, students use the data to graph functions describing
   each student’s movement.

2. Students may complete the *Equations of Attack* activity sheet from *NCTM
   Illuminations* (or an equivalent activity). The activity consists of students
   working in pairs to take turns finding lines (representing cannon shots) to
   connect from one point (their cannon) to another point (their opponent’s
   battleship). Once one of the students finds all the lines that properly connect
   their cannons to their opponents’ battleships, that student wins the
   competition. Each student places five battleships on lattice points within the
   first quadrant, and students are assigned five alternating positions on the
   positive y-axis. Since the cannon positions are on the y-axis, students must
determine the necessary slope to connect a line from their cannon to their
   opponent’s battleship. To differentiate instruction, the cannon and battleship
   positions may be placed throughout all four quadrants within the coordinate
   plane, making the task of finding the line of attack more difficult.
**Differentiated Instruction (For Learners Needing More Help)**

Maintain students’ note cards and the class bulletin board. Provide directions on how to graph a function in slope-intercept form.

When doing a jigsaw puzzle style of group work, carefully assign problems that suit students’ capabilities.

**Differentiated Instruction (Enrichment)**

Challenge students to graph an equation such as $y = -\frac{5}{3}x - 8$ on a 10-by-10 coordinate plane centered at the origin. Students will likely interpret the slope as “down 5, right 3”, requiring them to “go off” the bottom of the graph unless they interpret the slope as “up 5, left 3”.

Give students equations with decimal numbers and non-integer intercepts to graph.

**Journal Prompt**

You may have heard that “two points determine a line”. What does this mean? Based on your work with linear equations, what other two pieces of information determine a line? Explain.

**Resources and Materials**

- Activity 4.4.1 Effects of Changing Parameters
- Activity 4.4.2 Slope-Intercept Form
- Activity 4.4.3 Practice with Slope-Intercept Form
- Activity 4.4.4 Making a Profit
- Activity 4.4.5 Applications of Slope-Intercept Form
- Activity 4.4.6 Parallel and Perpendicular Lines
- Activity 4.4.7 More Parallel and Perpendicular Lines
- Exit Slips 4.4.1 and 4.4.2
- Straight Edges for drawing linear graphs
- Bulletin Board for key concepts
- Graphing Calculators
Effects of Changing Parameters

In this activity, you will learn how the parameters (numbers) $m$ and $b$ affect a linear function in the form $y = mx + b$. The form $y = mx + b$ is known as slope-intercept form.

**Instructions:**

We will use our graphing calculators to explore linear functions. First, we need a good window. The window controls the range of $x$ and $y$ values displayed on the graphing calculator. We will use a window where the $x$-axis will go from negative five to five, and the $y$-axis will go from negative five to five. To do this:

a. Turn the calculator **ON**.
b. Press the **WINDOW** button.
c. In the **WINDOW** menu, set $X_{	ext{min}} = -5$, $X_{	ext{max}} = 5$, $X_{	ext{scl}} = 1$, $Y_{	ext{min}} = -5$, $Y_{	ext{max}} = 5$, $Y_{	ext{scl}} = 1$
d. Enter the function: $y = 1x + 0$ into the graphing calculator. To do this:
   • Press the $Y= $ button.
   • Enter your equation into $Y_{1}=$. For the $x$-variable, the button is the $X,T,\Theta,n$ button.
e. Graph the function. To do this, press the **GRAPH** button.

1. Sketch the graph of $y = 1x + 0$.
   a. What is the slope?
   b. What is the value of $b$ in the equation?
   c. What is the $y$-intercept?
2. Graph \( y = 2x + 0 \) in the calculator and sketch the graph.
   
   a. What changed from the graph in question (1)?
   
   b. What is the slope?
   
   c. What is the value of \( b \) in the equation?
   
   d. What is the \( y \)-intercept?

3. Graph \( y = \frac{1}{2}x + 0 \) in the calculator and sketch the graph.
   
   a. What changed from the graph in question (1)?
   
   b. What is the slope?
   
   c. What is the value of \( b \) in the equation?
   
   d. What is the \( y \)-intercept?
4. If \( m \) is a positive number, what happens to the graph of a linear function as \( m \) increases?

5. If \( m \) is a positive number, what happens to the graph of a linear function when \( m \) decreases but remains positive?

6. Graph \( y = -1x + 0 \) in the calculator and sketch the graph.
   a. What changed from the graph in question (1)?
   b. What is the slope?
   c. What is the value of \( b \) in the equation?
   d. What is the \( y \)-intercept?

7. Graph \( y = -2x + 0 \) in the calculator and sketch the graph.
   a. What changed from the graph in question (6)?
   b. What is the slope?
   c. What is the value of \( b \) in the equation?
   d. What is the \( y \)-intercept?
8. Graph \( y = -\frac{1}{2}x + 0 \) in the calculator and sketch the graph.

   a. What changed from the graph in question (6)?

   b. What is the slope?

   c. What is the value of \( b \) in the equation?

   d. What is the \( y \)-intercept?

9. What happens to the graph of a linear function when \( m \) is negative?

10. When \( m \) is negative, describe how you can change \( m \) to make the line steeper, and how you can change \( m \) to make the line flatter.

11. What point do all of the lines you have graphed have in common?

12. Does changing \( m \) have any effect on the \( y \)-intercept of the graph?

13. Predict what the graph will look like when \( m=0 \).

14. Test your prediction by graphing a line with \( m=0 \) on your calculator. Was your prediction correct? If not, what kind of line did you get?

15. In your own words, describe the value of \( m \)’s overall effect on the graph of a line.
16. Graph \( y = 1x + 0 \) in the calculator and sketch the graph.

   a. What is the slope?

   b. What is the value of \( b \) in the equation?

   c. What is the \( y \)-intercept?

17. Graph \( y = 1x + 2 \) in the calculator and sketch the graph.

   a. What changed from the graph in question (16)?

   b. What is the slope?

   c. What is the value of \( b \) in the equation?

   d. What is the \( y \)-intercept?

18. Graph \( y = 1x + 4 \) in the calculator and sketch the graph.

   a. What changed from the graph in question (16)?

   b. What is the slope?

   c. What is the value of \( b \) in the equation?

   d. What is the \( y \)-intercept?
19. Graph $y = 1x - 2$ in the calculator and sketch the graph.

   a. What changed from the graph in question (16)?
   
   b. What is the slope?
   
   c. What is the value of $b$ in the equation?
   
   d. What is the $y$-intercept?

20. If $b$ has a negative value then the $y$-intercept is (above, below) the x-axis. Circle one answer.

21. If $b$ has a positive value then the $y$-intercept is (above, below) the x-axis. Circle one answer.

22. What is the $y$-intercept of the equation $y = 2x + 4$?

23. What is the $y$-intercept of the equation $y = x - 5$?

24. How does changing $b$ in a linear function affect the graph? Be as specific as possible.
Slope-Intercept Form of a Line

1. Fill in the table for each function. Plot the points from the tables on the coordinate planes. Use a ruler or straightedge to draw a line through the points.

   (a) \( y = \frac{1}{2}x + 2 \)

   \[
   \begin{array}{c|c|c|c|c|c}
   x & -2 & -1 & 0 & 1 & 2 \\
   \hline
   y & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
   \end{array}
   \]

   (b) \( y = -2x - 3 \)

   \[
   \begin{array}{c|c|c|c|c|c}
   x & -2 & -1 & 0 & 1 & 2 \\
   \hline
   y & \text{ } & \text{ } & \text{ } & \text{ } & \text{ } \\
   \end{array}
   \]

2. Calculate the slope \( (m) \) of each function in problem 1. You may either find the rise and run directly from the graph or use the slope formula to get your answer. Leave your answer as a fraction in simplest form. *Hint: Pick 2 easy points from each line to work with.*

   Line (a) \( m = \)  
   Line (b) \( m = \)

3. Do the slopes you just calculated appear anywhere in the functions from problem 1? If so, where do they appear?

4. Find the \( y \)-intercept of each function. *Remember:* the \( y \)-intercept is the point where the line crosses the \( y \)-axis.

   Line (a) \( y \)-intercept:  
   Line (b) \( y \)-intercept:
5. Do the $y$-intercepts you just found appear anywhere in the functions from problem 1? If so, where do they appear?

6. Using what you’ve just discovered, can you think of an easier way to graph the lines from problem 1 on the first page without making an $x$-$y$ table? For example, explain how you would graph the function $y = \frac{1}{2}x + 2$?

The linear functions on the first page are written in a special form called **slope-intercept form**. When a linear function is in the form $y = mx + b$ the slope of the line is $m$ and the $y$-intercept is $b$.

The number in front of the $x$ variable, $m$, is the coefficient of $x$ and is the slope. The constant term is the $y$-intercept.

$x$ and $y$ are the **variables**. For any given example, $x$ and $y$ can vary. You can choose any real number for $x$ and find the corresponding $y$ value.

$m$ and $b$ are **parameters**. For any given example, the specific numbers for $m$ and $b$ are fixed. The given values for $m$ and $b$ will not change for that particular equation.
7. For each function, find the slope \( m \) and \( y \)-intercept \( b \), and then graph each function.

(a) \( y = \frac{-2}{7}x + 4 \): \( m = \quad b = \quad \)

(b) \( y = \frac{5}{3}x - 7 \): \( m = \quad b = \quad \)

(c) \( y = 3x \): \( m = \quad b = \quad \)

(d) \( y = \frac{-5}{4}x - 4 \): \( m = \quad b = \quad \)
8. For each line, find the slope \((m)\) and \(y\)-intercept \((b)\), and then write the equation of the line in slope-intercept \((y = mx + b)\) form.

Slope \((m)\):_____________________

\(y\)-intercept \((b)\):__________________

Equation:_______________________
Practice with Slope-Intercept Form

\[ y = mx + b \] is the slope-intercept form of an equation: \( m \) is the slope and \( b \) is the \( y \)-intercept.

**Method for Graphing a Function in Slope-Intercept Form**

- Start by plotting the point \((0, b)\) which is the \( y \)-intercept.
- From that point, count the rise and run according to slope to obtain other points.
- Connect the points with a line.
- Do a quick check to see that your graph makes sense. If the slope is negative, is your line decreasing? If the slope is a number close to zero is your line relatively flat?

1. For each function find the slope, \( m \), and \( y \)-intercept, \( b \). Then graph each function. Label the coordinates \((x, y)\) of two points on your line.

(a) \( y = \frac{7}{2}x - 3 \quad m = \text{_____} \quad b = \text{_____} \)
(b) \( y = -\frac{1}{5}x - 2 \quad : \quad m = \text{_____} \quad b = \text{_____} \)
(c) \( y = -6x + 4 \quad m = \text{_____} \quad b = \text{_____} \)
(d) \( y = \frac{-5}{2}x + 3 \quad m = \text{_____} \quad b = \text{_____} \)
2. For each graph, find the slope, \( m \), and \( y \)-intercept, \( b \), and then write the equation of the line in slope-intercept \((y = mx + b)\) form.

(a)  
(b)  

Slope \((m)\): _____ \(y\)-intercept \((b)\) ______

Equation: __________________________

Slope \((m)\): _____ \(y\)-intercept \((b)\) ______

Equation: __________________________
Making a Profit

1. At Eisenhower High School’s home games, booster clubs will often hold raffles to raise money for the team or for special causes. The Girl’s Basketball Team had a gift basket filled with movie items to be raffled off at the Coaches Versus Cancer night. The girls decided to charge $2 a raffle ticket.

   The gift basket contained: 4 tickets which cost $10.25 each, 12 boxes of candy which cost $2.50 each, a box of popcorn which costs $12, and 4 pairs of 3-D glasses which cost $4.25 each.

   a. Revenue is the amount of money the girls’ basketball team takes in – its income. In this case, revenue is a function of number of tickets sold. Let x be the number of tickets sold. What is the revenue function?

   b. Cost is how much money the girls’ basketball team spent. In this situation, cost is a constant function. What is the cost function? Hint: The cost function equals the expenses.

   c. Profit = Revenue – Cost. What is the profit function? (use the letter y for profit)

   d. What is the y-intercept of the profit function? Include units.

   e. What is the slope of the profit function? Include units. Remember slope is a rate of change.
2. Study the following window and graph. Number the axes.

   a. Does it match your profit function? How do you know?

   [Image of a graph]

   b. ‘Breaking even’ means that the business neither loses nor gains any money. What number can you substitute in for \( y \) in the profit function to represent breaking even?

   c. How many tickets would the team have to sell to break even?

3. The team decided that they would like to make more profit for fighting cancer. Study the graph to the right showing the team’s new plan. Number the axes.

   a. How are the graphs the same? How are the graphs different?

   [Image of a graph]

   b. Which parameter (slope or \( y \)-intercept) changed in their profit function? How do you know?
c. What does that tell you about the raffle’s ticket prices and operating costs? In other words, what did the team change about the raffle?

4. How else could the girls have raised more money? Would this be a change in the y-intercept or in the slope? How do you know?

5. The team decided that they would try another idea to make a greater profit for fighting cancer. Study the following graph the team came up with. Number the axes.

   a. How are this graph and the original graph the same?

   b. How are the graphs different?

   c. Which parameter (slope or the y-intercept) must have changed in their profit function?

   d. How do you know?

   e. What does that tell you about the raffle’s ticket prices and operating costs? In other words, what did the team change about the raffle? If the team made this change, what must have happened?
Applications of Slope-Intercept Form

During one of the great snowstorms of 2011, snow fell for nine hours at a rate of one-half inch per hour. Before the storm began, there were already six inches of snow on the ground.

1. What is the independent variable?

2. What is the dependent variable?

3. Write a heading for each column and then complete the table below.

<table>
<thead>
<tr>
<th>0</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

4. Does this situation create and increasing or decreasing function? Explain why.

5. Will the slope of this function be positive or negative? Explain why.
6. a. Make a graph: Plot the points from your table on the following coordinate plane. Make sure to scale and label your axes. Connect the points with a straight line using a ruler.

b. Calculate the slope of the function graphically, or by using the slope formula.

c. What is the real-world meaning of your value of the slope?

d. Find the y-intercept of the function. What is the real-world meaning of your y-intercept?

e. Write the equation for the line in slope-intercept (\( y = mx + b \)) form.
7. At some schools in Connecticut school is automatically canceled if there is a foot or more of snow on the ground. Was school canceled the day of this storm (remember, it snowed for a total of nine hours)? Show your work to explain your reasoning.

8. Let’s say the weatherman’s prediction was wrong, and it stopped snowing after 5 hours. How much snow would be on the ground? Show your work to explain your reasoning.

9. Again, let’s say the weatherman’s prediction was wrong, and the storm was worse than predicted. Snow actually fell at the rate of two inches per hour. How would this change your graph? Write a new equation to model this situation.
10. Answer the following questions using the table below based on the Teddy Bear Sale problem from Activity 4.2.7.

<table>
<thead>
<tr>
<th># of bears sold</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1500</td>
</tr>
<tr>
<td>50</td>
<td>-900</td>
</tr>
<tr>
<td>100</td>
<td>-300</td>
</tr>
<tr>
<td>150</td>
<td>300</td>
</tr>
<tr>
<td>200</td>
<td>900</td>
</tr>
<tr>
<td>250</td>
<td>1500</td>
</tr>
<tr>
<td>300</td>
<td>2100</td>
</tr>
</tbody>
</table>

a. Find the slope of this function. (Make sure to label your answer)

b. Find the \( y \)-intercept. What does this point represent?

c. Write the equation in \( y = mx + b \) form.

d. How many bears must you sell to break even (i.e. make a profit of 0)? Explain how you got your answer.
11. For each of the following situations, identify the slope (use units of measure) and y-intercept, state whether the function is increasing, decreasing, or neither, and write the equation that models the function in $y = mx + b$ form.

a. You open a new bank account with $50. Each month you deposit $10 into the account. You want to find an equation for the amount of money in your account as a function of time.

$m=_______$  $b=_________
Increasing/Decreasing/Neither
Equation:_____________________________________

b. You are scuba diving off the coast of Florida. Starting at the surface, each minute you descend seven feet into the ocean. What is your distance from the surface as a function of time?

$m=_______$  $b=_________
Increasing/Decreasing/Neither
Equation:_____________________________________

c. You start 4 meters away from the motion detector and do not move. What is your distance from the motion detector?

$m=_______$  $b=_________
Increasing/Decreasing/Neither
Equation:_____________________________________

d. At the start of your trip to Washington D.C., you completely fill your 15 gallon gas tank. Your vehicle averages 26 miles per gallon. The amount of gas in your tank is a function of the miles you drive.

$m=_______$  $b=_________
Increasing/Decreasing/Neither
Equation:_____________________________________

e. At 4 a.m. the temperature outside is -5 degrees Celsius. Every hour the temperature rises by 4 degrees. What is the temperature as a function of time?

$m=_______$  $b=_________
Increasing/Decreasing/Neither
Equation:_____________________________________

f. You are participating in the relay for life walkathon to raise money for cancer research. Your parents agree to pledge $25 regardless of the number of miles you walk. What is the amount of money raised?

\[ m = _____ \quad b = _____ \quad \text{Increasing/Decreasing/Neither} \]

Equation: ________________________________

g. Your club sells popcorn at the hockey games as a fundraiser. It costs you $8 for supplies. You sell each bag of popcorn for $0.75. What is your profit as a function of bags sold?

\[ m = _____ \quad b = _____ \quad \text{Increasing/Decreasing/Neither} \]

Equation: ________________________________

12. Ken wants to buy the new $270 game system, but he had only $15. Each week he saved the same amount of money. The table below shows how much money he will have each week. How many weeks will it take for Ken to have enough money to purchase his game system?

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount saved</td>
<td>15</td>
<td>22</td>
<td>29</td>
<td>36</td>
<td>42</td>
<td>49</td>
<td>56</td>
</tr>
</tbody>
</table>

a. Write an equation in slope intercept form that describes amount saved as a function of the number of weeks.

b. What is the real life meaning of slope in this situation?

c. What is the real life meaning of the y-intercept?

d. Solve an inequality or an equation to find how many weeks it will take Ken to save enough money for the game system.
13. An algebra teacher used the motion detector to show his pre-calculus students what the algebra 1 students are learning. He told a student to start 15 feet from the motion detector and walk at a steady pace of 2 feet per second toward the motion detector. The Pre-Calculus students could not figure out an equation for the distance-time graph that the motion detector showed.

a. Please tell them the equation in slope-intercept form.

b. Explain to them what the slope means in this situation.

c. What does the y-intercept mean in this situation?
Parallel & Perpendicular Lines

1. Jason and Scott plan on biking to the center of town to get ice cream at the convenience store. Since Scott had to put air in his tires, Jason was able to get 1 mile ahead of Scott before Scott left the house. Both bikers rode at a speed of 15 miles per hour.

   a. Write an equation in \( y = mx + b \) form that represents Jason’s trip.

   b. Write an equation in \( y = mx + b \) form that represents Scott’s trip.

   c. Will Jason and Scott meet before they both reach the store? Explain.

   d. If you were to graph both lines on the same coordinate plane, predict what your graph would look like.

2. a. On the same set of axes graph the following lines. Use a ruler to make your lines.

   \[
   y = 3x + 4 \\
   y = 3x - 4 \\
   y = 3x
   \]
b. What do you notice about the lines you graphed in problem 2?

c. What can you say about the slopes of parallel lines?

3. a. On the same set of axes graph the following lines. Use a ruler to make your lines.

\[ y = 2x + 1 \]
\[ y = -\frac{1}{2}x - 1 \]

b. How does the line \( y = 2x + 1 \) compare to \( y = -\frac{1}{2}x - 1 \)? Discuss the slopes of each line and what type of angle is formed where the lines intersect.
4. a. On the same set of axes graph the following lines. Use a ruler to make your lines.

\[ y = -\frac{2}{3}x + 4 \]

\[ y = \frac{3}{2}x - 3 \]

b. How does the line \( y = -\frac{2}{3}x + 4 \) compare to \( y = \frac{3}{2}x - 3 \)? Discuss the slopes of each line and what type of angle is formed where the lines intersect.

**Perpendicular lines** have slopes that are *opposite reciprocals* of each other.

For example, what is the opposite reciprocal of 6?

Answer:

**Step 1 →** Take the 6 and write it as a fraction \( \frac{6}{1} \)

**Step 2 →** Flip it over (also known as the reciprocal) \( \frac{1}{6} \)

**Step 3 →** Change the sign (also known as the opposite) \( -\frac{1}{6} \)

Therefore, the opposite reciprocal of 6 is \( -\frac{1}{6} \).

5. Find the opposite reciprocal of each number.

a. -4  
   b. \( \frac{5}{2} \)  
   c. \( -\frac{3}{7} \)  
   d. 0
6. Below are the equations of four lines.

\[ y = -\frac{1}{5}x + 7 \quad y = \frac{1}{5}x - 8 \quad y = -5x - 6 \quad y = -\frac{1}{5}x + 8 \]

a. Which pairs of lines are parallel?

b. Which pairs of lines are perpendicular?

7. Write the equation of a line parallel to the given line. Note: There is more than one correct answer.

a. \( y = 4x + 1 \) is parallel to

b. \( y = -9x - 2 \) is parallel to

c. \( y = -\frac{5}{6}x + 2 \) is parallel to

d. \( y = \frac{1}{2}x - 11 \) is parallel to

8. Write the equation of a line perpendicular to the given line. Note: There is more than one correct answer.

a. \( y = -\frac{4}{3}x + 1 \) is perpendicular to

b. \( y = 9x + 13 \) is perpendicular to

c. \( y = \frac{1}{7}x - 6 \) is perpendicular to

d. \( y = -\frac{3}{7}x - 4 \) is perpendicular to
9. Summarize what you have learned in this activity. Fill in the blank of each statement or circle the appropriate word to make the statement true.

   a. Parallel lines have the _________________________ slope.

   b. Parallel lines have (same, different) y-intercepts.

   c. Parallel lines _______________________________ intersect each other.

   d. Two lines with slopes that are ______________________________ reciprocals of each other are perpendicular to each other.

   e. Perpendicular lines have (same/different) y-intercepts.

   f. A pair of perpendicular lines intersects at _________________________ angle.
More Parallel & Perpendicular Lines

**Parallel lines** have the same slope, but different \( y \)-intercepts.
**Perpendicular lines** have slopes that are opposite reciprocals.
**Horizontal lines** have slopes equal to zero.
**Vertical lines** have slopes that are undefined.

1. Find the slope of the line passing through the two points and describe the line as increasing, decreasing, horizontal, or vertical.
   a) \((2, 1); (4, 5)\)
   b) \((-1, 0); (3, -5)\)
   c) \((2, 1); (-3, 1)\)
   d) \((-1, 2); (-1, -5)\)

2. Determine whether the graphs of each pair of equations are parallel, perpendicular, or neither.
   a) \(y = 3x + 4; y = 3x + 7\)
   b) \(y = -4x + 1; \ y = \frac{1}{4}x + 3\)
   c) \(y = 2x - 5; y = 5x - 5\)
   d) \(y = -\frac{1}{3}x + 2; \ y = 3x - 5\)
   e) \(y = \frac{3}{5}x - 3; \ y = -\frac{3}{5}x - 2\)
   f) \(y = 4; \ y = 17\)
   g) \(y = 7x + 2; \ y = \frac{-1}{7}x + \frac{8}{7}\)
   h) \(y = 1; \ x = -4\)
3. Find an equation of a line parallel to and perpendicular to:

a) \( y = -5x + 4 \)  
Parallel:  
Perpendicular: 

b) \( y = \frac{2}{3}x - 7 \)  
Parallel:  
Perpendicular: 

c) \( y = -2 \)  
Parallel:  
Perpendicular:

4. Regina starts driving 3 miles northwest of Norwich. She is heading toward Hartford along Route 2. At the same time Dave starts driving 1 mile northwest of Norwich also heading toward Hartford along Route 2. Both Regina and Dave drive at 60 miles per hour. Let miles from Norwich be the dependent variable.

a. Write a distance-time function for Regina.

b. Write a distance-time function for Dave.

c. If graphed, would the lines for these equations be parallel, perpendicular, or neither?

d. Will Dave and Regina ever meet on the way to Hartford?
5. Use the graph below to answer the following questions.

![Graph of a line](image)

a. What is the $y$-intercept of the line?

b. What is the slope of the line?

c. Write the equation of the line in $y = mx + b$ form.

d. Draw a line parallel to this line on the coordinate plane. Write the equation of your parallel line.

e. Draw a line perpendicular to this line on the coordinate plane. Write the equation of your perpendicular line.
Unit 4: Investigation 5 (4 Days)

FORMS OF LINEAR EQUATIONS

**CCSS:** F-LE5, F-LE2, F-LE1

**Overview**
Students learn about direct variation \( y = kx \) as an equation in slope-intercept form that has a \( y \)-intercept of zero. Students are introduced to the standard form of linear equations, rewrite equations in standard form into slope-intercept form, and use linear models in various forms to explore real world situations.

**Assessment Activities**

**Evidence of Success: What Will Students Be Able to Do?**
- Recognize two forms of a linear equation: standard form and slope-intercept form.
- Recognize direct variation problems as a special case of slope-intercept form, with constant of proportionality equal to \( y/x \), the slope and a \( y \)-intercept of 0.
- Model a real world situation with an appropriate form of a linear equation.
- Find \( x \)- and \( y \)-intercepts and slope of a linear function given any form of the equation.
- Draw the graph given the \( x \)- and \( y \)-intercepts, slope and \( y \)-intercept.
- Explain what the \( x \)- and \( y \)-intercepts represent in the context of a real world problem.
- Transform linear equations from standard form to slope-intercept form.

**Assessment Strategies: How Will They Show What They Know?**

**Exit Slip 4.5** asks students to transform an equation from standard form to slope-intercept form and to graph an equation using two intercepts.

**Unit 4 Investigation 5 Quiz** assesses student understanding of both direct variation and standard form.

**Journal Entry 1** assesses student understanding of when direct variation is a suitable model.

**Journal Entry 2** asks students to describe the advantages and disadvantages of two forms of the linear equation.

**Journal Entry 3** asks students to describe real world situations modeled by standard and slope-intercept forms.

**Launch Notes**
Discuss the designing and building trades with students. You can show a You Tube video “Gang Setting Trusses” [http://www.youtube.com/watch?v=FbQxN7iv-ns&feature=related](http://www.youtube.com/watch?v=FbQxN7iv-ns&feature=related) to generate interest. Then have students use a ruler to draw out a scale model of a simplified roof truss where vertical posts are placed along the triangular face. Ask how many triangles they see in their drawing. With a ruler, have students measure the distance a post is from the corner eave and the height of the post. Ask probing questions to elicit students’ observations about similar triangles and proportionality.
Closure Notes
Students should be very comfortable using standard form and slope-intercept form to model and explore real world situations. They should be able to identify the advantages of each form, be able to choose the appropriate form, and recognize the equivalence of the forms.

Teaching Strategies

I. In Activity 4.5.1 Direct Variation, students explore direct variation problems in real world situations. Students should recognize that direct variation occurs when a linear function has a $y$-intercept of 0 and that a direct variation equation in the form $y = kx$ is a special case of the slope-intercept form. Students may work in small groups or pairs to construct and graph direct variation functions.

In addition to the applications included in Activity 4.5.1, you may choose to present or discuss the following examples of direct variation:

(a) Model the cost to put gasoline that costs $2.89/gallon in a car based on the number of gallons of gasoline purchased.
(b) Find the amount of drug that a dog needs if it should vary with the weight of the dog.
(c) Find the total tutoring bill if a tutor charges $45 per hour.
(d) Revisit some distance, rate and time problems.

Through exploration, students will discover how to determine whether or not a linear function scenario is a direct variation scenario. Assign Activity 4.5.2 More Direct Variation for more practice.

Journal Entry 1

Which of the following two scenarios is direct variation and which is not? Explain your answer.

A. The plumber charges $60 per hour and a $75 fee for the house call.
B. The auto repair shop charges $80 per hour to fix my car.
II. Next, introduce the standard form of linear equations. The standard form lends itself well to situations when a constant value results from the combination of two rates, such as the rate charged for a child’s ticket and the rate charged for an adult ticket combined. One example is to ask students what combinations of $3 children tickets and $5 adult tickets need to be sold to make $90. Students may like guessing coin values: what combinations of dimes and quarters will total $2? Discuss discrete versus continuous graphs: one can’t sell 1.5 tickets, but if you combine cashews at $8 a pound with peanuts at $3 a pound the function is continuous.

Students should practice finding the intercepts of a line in standard form by solving for $x$ when $y = 0$ and solving for $y$ when $x = 0$. Students will then learn how to use the intercept graphing method as a method for determining the slope of a line by calculating the ratio of the rise and the run between the intercepts. Students should then practice transforming equations from standard form to slope-intercept form and then graph the equation. Graphical representations and algebraic transformations highlight the mathematical equivalence of an equation regardless of its form. See Activity 4.5.3 Standard Form of a Linear Equation. You can assess understanding of these outcomes with Exit Slip 4.5.

Continue to practice problems in standard form mixed with problems that lend themselves to slope-intercept form. Have students articulate the advantages of each form in a journal prompt. See Activity 4.5.4 More Standard Form and Activity 4.5.5 Practice with Standard Form and Slope-Intercept Form.

At the end of this investigation you may give students Unit 4 Investigation 5 Quiz.

**Journal Entry 2**

What are the advantages of graphing an equation in slope-intercept form, and the advantages of graphing an equation in standard form?

**Differentiated Instruction (For Learners Needing More Help)**

Maintain students’ note cards and the class bulletin board. Include directions on how to find the constant of variation and how to graph a function in standard form by finding intercepts. A table with 0 filled in for $x$ on the first row and 0 filled in for $y$ on the second row is a visual reminder for students to substitute 0 in for $x$ and solve for $y$, and substitute 0 in for $y$ and solve for $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>0</td>
</tr>
</tbody>
</table>
Differentiated Instruction (Enrichment)

Ask students to research the “rule of thumb” for estimating the distance a person is from lightning based on the time lapse between when a person sees the lightning bolt and hears the thunder. Ask students to justify why this rule works.

Group Activity

Activity 4.5.5 Practice with Standard Form and Slope-Intercept Form lends itself well to group problem solving since the problems have little scaffolding and the students have to choose which form of a linear equation to use.

Journal Prompt 3

When modeling a real life situation, when do you use slope-intercept form and when do you use standard form?

Resources and Materials

- Activity 4.5.1 Direct Variation
- Activity 4.5.2 More Direct Variation
- Activity 4.5.3 Standard Form of a Linear Equation
- Activity 4.5.4 More Standard Form
- Activity 4.5.5 Practice with Standard Form and Slope-Intercept Form
- Exit Slip 4.5
- Unit 4 Investigation 5 Quiz
- Rulers
- Bulletin Board for key concepts
- Student Journals
- Graphing Calculators
- Video on roof trusses [http://www.youtube.com/watch?v=FbQxN7iv-ns&feature=related](http://www.youtube.com/watch?v=FbQxN7iv-ns&feature=related)
Direct Variation

1. When building a roof, carpenters place posts every 2 feet along the horizontal support beam starting at the eave. The diagram below illustrates this.

![Diagram of roof with posts every 2 feet from eave to peak.](image)

a. Complete the table below.

<table>
<thead>
<tr>
<th>Post</th>
<th>Horizontal distance from post to eave ((x))</th>
<th>Height of Post ((y))</th>
<th>Ratio (y \div x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1.5</td>
<td>0.75</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Circle the appropriate words to make the statement true. Consider only the posts from the left eave to the center peak.

As the height of post (increases/decreases), the horizontal distance from post to eave (increases/decreases).

c. What do you notice about the last column? Does it represent anything we learned about equations of lines? If so, what?

d. One of the points is \((0,0)\). What would this point mean in the context of the problem?

e. Use the slope formula to find the slope between the point \((0,0)\) and \((4,3)\).
f. Use the slope formula to find the slope between the two points \((0, 0)\) and \((2, 1.5)\).

g. Describe how you can find the slope in a direct variation problem.

h. Write a linear equation in \(y = mx + b\) form for the height of a post as a function of distance from eave.

i. Graph the linear equation. Label the axes and choose a proper scale.
2. A bicyclist traveled at a constant speed during a timed practice period. His distance varied directly with the time elapsed. After 10 minutes elapsed, his distance travelled was 3 miles.

a. Complete the table below.

<table>
<thead>
<tr>
<th>Elapsed Time (x) in minutes</th>
<th>Distance (y) in miles</th>
<th>Ratio $\frac{y}{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3</td>
<td>.3</td>
</tr>
<tr>
<td>20</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Circle the appropriate words to make this statement true.

As the elapsed time (increases/decreases), the distance in miles (increases/decreases).

c. Does any value in the table represent the slope? If so, what?

d. One of the points is (0,0). What does this point mean in the context of the problem?

e. Use the slope formula to find the slope between the two points (0,0) and (10,3).

f. Use the slope formula to find the slope between the two points (0,0) and (20,6).

g. Write a linear equation in $y = mx + b$ form.
3. When two variable quantities have a constant ratio, their relationship is called a *direct variation*. It is said that one variable “varies directly” with the other variable.

Fill in the blanks.

a. In the roof problem (problem 1) ______________________ varies directly with ______________________.

b. In the bike problem (problem 2) ______________________ varies directly with ______________________.

4. The constant ratio (also known as the slope) is called the *constant of variation*.

a. Looking back at the roof problem (problem 1), what was the constant of variation?

b. Looking back at bike problem (problem 2), what was the constant of variation?

5. Describe how to find the constant of variation (slope).
6. We can also say that \( y \) is proportional to \( x \), which means the same as \( y \) varies directly with \( x \). The \( x \) multiplied by the constant of variation will equal \( y \).

   a. In the roof problem 1, you multiply the length from the eave by .75 to get ________.

   b. In the bike problem 2, you multiply the elapsed time by ______ to get __________.

   c. In the roof problem, we could set up a proportions such as: \( \frac{1.5}{2} = \frac{3}{4} \). Write a proportion for the bike problem.

7. The formula often used for direct variation is \( y = kx \), where \( k \) is the constant of variation. A direct variation equation is a special case of the slope-intercept form. We can rewrite it as \( y = mx \).

   a. What was the linear equation you found in problem 1?

   b. What is the \( y \)-intercept of this equation?

   c. What was the linear equation you found in problem 2?

   d. What is the \( y \)-intercept of this equation?

8. In the roof problem, what special relationship exists among the 4 nested triangles formed by the roof line, the 4 different posts and the base?
9. What is an example of a direct variation problem compared to one that is not? Here are two scenarios, the first is direct variation, and the second is not.

**Scenario 1:** The cost of topsoil varies directly as the number of cubic yards purchased if you can pick it up in your own truck. 5 cubic yards costs $60.

Since cost varies directly as number of cubic yards, then cost is a function of cubic yards.

a. What is the independent variable $x$?

b. What is the dependent variable $y$?

c. What is the constant of variation $y/x$?

d. Write an equation for cost as a function of cubic yards purchased.

**Scenario 2:** If you have the company deliver it in their truck they charge a $50 delivery fee in addition to the $12 per cubic yard.

a. Write an equation for cost as a function of cubic yards purchased for customers that have the soil delivered.

b. Make a table of values for each situation, showing how much it costs in total for 1-6 cubic yards:

<table>
<thead>
<tr>
<th>Customers that haul their own topsoil</th>
<th>Customers that have the topsoil delivered</th>
</tr>
</thead>
<tbody>
<tr>
<td># cubic yards</td>
<td>Cost (in $)</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
c. When you double the amount of topsoil you buy from 2 yards to 4 cubic yards, is the cost also doubled?

<table>
<thead>
<tr>
<th>Customers that haul their own</th>
<th>Customers that have the topsoil delivered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost is /is not doubled when amount of soil is doubled.</td>
<td>Cost is /is not doubled when amount of soil is doubled.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Customers that haul their own</th>
<th>Customers that have the topsoil delivered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes or No?</td>
<td>Yes or No?</td>
</tr>
</tbody>
</table>

**Conclusion:** In direct variation problems, if you multiply the independent variable by a number, the dependent variable is multiplied by the same number. For example, if you double the input, the output will double. In direct variation problems, the output is proportional to the input.

If the \( y \)-intercept is not zero, the linear relation is not a direct variation problem. Doubling the input will not result in doubling the output. The output is not a constant multiple of the input.
More Direct Variation

**Direct variation** is represented by a linear function that has a y-intercept of 0. Direct variation is represented by a linear graph that goes through the origin. In words, we say “y varies directly with x” or “y is proportional to x”. The ratio of y/x is the constant of variation, which is the same as the slope. For example: if y is 3 times as big as x, then y=3x. Notice that slope is y/x = 3.

Examples of direct variation include:
1. \( y = 3x \)
2. \( y = -5x \)
3. \( y = \frac{1}{2} x \)
4. \( y = x \)
5. \( 5x + 2y = 0 \)

Non-Examples of direct variation include:
1. \( y = 3x + 2 \)
2. \( y = -6x - 3 \)
3. \( y = \frac{1}{4} x + 3 \)
4. \( y = x - 3 \)

1. Circle the examples that represent direct variation.
   a. \( y = \frac{1}{5} x \)
   b. \( y = 3x - 4 \)
   c. \( y = x + 1 \)
   d. \( y = 2x \)

2. Your distance from a lightning strike varies directly with the time it takes you to hear thunder. If you hear thunder 10 seconds after you see lightning, you are about 2 miles from the lightning.
   a. What is the independent variable?
   b. What is the dependent variable?
c. What is the constant of variation?

d. Write an equation for distance as a function of time.

e. How many miles away is a lightning bolt if you hear the thunder 5 seconds after you see the lightning?

3. A bicyclist traveled at a constant speed during a timed practice period. Write an equation and find the distance the cyclist traveled in 30 minutes.

<table>
<thead>
<tr>
<th>Elapsed time (minutes)</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>25</td>
<td>7.5</td>
</tr>
</tbody>
</table>

4. Gasoline is now $3.80 per gallon. Write an equation to model the total cost to put gasoline in a car based on the number of gallons of gasoline purchased.

5. The amount of blood in a person’s body varies directly with body weight. A person who weighs 160 pounds has about 5 quarts of blood. Write an equation to model this situation.
Standard Form of a Linear Equation

\[ Ax + By = C \]

- The slope-intercept form of a line is \( y = mx + b \).
- The standard form of a line is \( Ax + By = C \).
- The parameters \( A, B \) and \( C \) are may be any real numbers. Often they are integers.
- You have used the standard form of an equation with you investigated iPods and points scored at basketball games.
- You can transform an equation in standard form into slope-intercept form by solving for \( y \).
- You can transform an equation in slope-intercept form into standard form by getting the constant term alone on the right side of the equal sign. Multiply the equation by a common denominator if necessary to clear fractions.
- If you multiply or divide both sides of an equation in standard form by the same number, you’ll get an equivalent equation that is also in standard form.

Here is an example of a linear function that lends itself well to slope-intercept form:

It costs $200 to open the St Vincent DePaul Soup for the evening meal regardless of the number of people served. The average meal cost per person is $4. What is the total cost of feeding people? Let \( x \) be the number of people who eat, and let \( y \) be the total cost.

An equation to model this situation is \( y = 4x + 200 \).

Here is an example of linear function that lends itself well to standard form:

Each year, Norwich students make hundreds of clay soup bowls for a soup kitchen fundraiser known as the “Empty Bowls Project”. Then the public is invited to buy a meal of soup, bread and cookies and receive a beautiful handmade bowl. Musicians volunteer to play for the dinner. For $12 the customers choose a bowl to keep and get the simple meal. For $8, you get a simple meal, but no handmade bowl. Last year, $960 was raised for charity. What are the possible combinations of meal-alone tickets and bowl-meal tickets that were sold at Empty Bowls? Let \( x \) represent the number of bowl-meal tickets sold. Let \( y \) represent the number of meal-alone tickets sold.

An equation that models the situation is \( 12x + 8y = 960 \).

1. Answer the following questions based on the equation \( 12x + 8y = 960 \).

a. What does the \( 12x \) mean?
b. What does the $8y$ mean?

c. The standard form of a linear equation is $Ax + By = C$. In $12x + 8y = 960$, what number is:

$A \underline{\quad} \quad B \underline{\quad} \quad C \underline{\quad}$

d. Fill in the table that gives possible combinations for the numbers of bowl-meal tickets and the number of meal-alone tickets that might have been sold to raise $960.$

<table>
<thead>
<tr>
<th>$x$: # of Bowl-Meals tickets at $12$ each</th>
<th>$y$: # of Meal Alone tickets at $8$ each</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

e. Write ordered pairs and graph the points on a coordinate plane.
f. What is the x-intercept? What does it mean in the context of the problem?

g. What is the y-intercept? What does it mean in the context of the problem?

h. What is the slope of the equation and what does it mean in the context of the problem?

i. Transform the equation $12x + 8y = 960$ into slope-intercept form ($y = mx + b$). Check to see that your answer in part (g) and (h) is the same slope and y-intercept in part (i).

2. Graph the linear functions in standard form by finding the x and y intercepts. Then find the slope and write the equation in slope-intercept form.
a. Given $3x + 2y = 12$, find both intercepts and the slope. Then draw the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$x$-intercept:

$y$-intercept:

Slope:

b. Write $3x + 2y = 12$ in slope-intercept form: $y = mx + b$.

c. From the slope-intercept form of the equation, the slope is _____ and the $y$-intercept is _______. Check to see that the results agree with your answers in part (a).

d. Given $4x - 5y = 20$, find both intercepts and the slope. Then draw the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$x$-intercept:
y-intercept:

Slope:

e. Write $4x - 5y = 20$ in slope-intercept form: $y = mx + b$.

f. From the slope-intercept form of the equation, the slope is _____ and the y-intercept is ______. Check to see that the results agree with your answers in part (d).
g. Given $6x - 5y = 30$, find both intercepts and the slope. Then draw the graph.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$x$-intercept:

$y$-intercept:

Slope:

h. Write $6x - 5y = 30$ in slope-intercept form: $y = mx + b$.

3. What is one advantage of the standard form (compared to slope-intercept form)?

4. What is one advantage of slope-intercept form?
5. For its community service this year, the GHS Honors Society has “adopted” the patients in the neighborhood Convalescent Home. The students visit the patients one afternoon a month to play cards, chat, read aloud and write letters for patients. When Jewett City Florist learned about the Honor Society’s good deeds, they donated 360 stems of flowers and greens to the Honors Society to bring cheer to the patients in the convalescent home. The students will make either a corsage for the women or a boutonniere for the men. Corsages will require 5 stems, and the boutonniere will require 3 stems. If the students use all 360 stems, what are the possible number of corsages and boutonnieres they can make?

a. Choose a variable to represent the number of corsages the students can make.

b. Choose a variable to represent the number of boutonnieres the students can make.

c. Fill in 5 rows of a table that shows a specific number of boutonnieres depending on the number of corsages made.

<table>
<thead>
<tr>
<th># of 5 stem corsages made</th>
<th># of 3 stem boutonnieres</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

d. Write a linear equation in standard form that models this situation.

e. Write the equation from part (d) in slope-intercept form.
More Standard Form

The standard form of a line is \( Ax + By = C \).
- The \( x \)'s and \( y \)'s are on the same side of the equal sign.
- The constant term is alone on the other side of the equal sign.
- To find the \( x \)-intercept, substitute 0 in for \( y \), and solve for \( x \).
- To find the \( y \)-intercept, substitute 0 in for \( x \) and solve for \( y \).
- To transform an equation from standard form to slope-intercept form, solve for \( y \).
- The slope-intercept form of a line is \( y = mx + b \).

1. Determine if the equation is in standard form. If not, explain why.
   a. \( 5x - y = 8 \)
   b. \( y = -4x - 3 \)
   c. \( x + 3y + 7 = -2 \)
   d. \( x + 3y = 0 \)
   e. \( y = 4 + 4(x - 2) \)
   f. \( -6x + 4y = -7 \)

2. Find the \( x \)-intercept and \( y \)-intercept.
   a. \( 5x + 2y = 10 \)  
   b. \( x - 3y = -6 \)
   c. \( -4x + y = 4 \)  
   d. \( 2x + 4y = 12 \)
3. Plot the intercepts from question 2 and connect them to make a line.

   a. $5x + 2y = 10$

   b. $x - 3y = -6$

   c. $-4x + y = 4$

   d. $2x + 4y = 12$

4. Use the graphs in question 3 to find the slope of each line.

   a. $5x + 2y = 10$  \(\text{Slope} = \) \\
   b. $x - 3y = -6$  \(\text{Slope} = \) \\
   c. $-4x + y = 4$  \(\text{Slope} = \) \\
   d. $2x + 4y = 12$  \(\text{Slope} = \)

5. Transform the following equations into slope-intercept form.

   a. $5x + 2y = 10$

   b. $x - 3y = -6$
c. \(-4x + y = 4\) 

6. You are in charge of buying the hamburger and chicken for a barbecue. The hamburger costs $2 per pound and the chicken costs $3 per pound. You have $30 to spend.

a. Use the verbal model below to help you write an equation that models the different amounts of hamburger and chicken that you can buy.

\[
\text{Price of Hamburger} \cdot \text{Weight of Hamburger} + \text{Price of Chicken} \cdot \text{Weight of Chicken} = \text{Spend}
\]

Let \(x\) represent the weight of hamburger (in pounds) and \(y\) represent the weight of chicken. Write an equation that models how much of each type of meat you can purchase for $30.

b. Find the \(x\)-intercept. What does it mean in the context of the problem?

c. Find the \(y\)-intercept. What does it mean in the context of the problem?
d. Graph using the intercepts. Make sure to label the axes and choose a proper scale.

![Graph](image)

e. If you buy 3 pounds of hamburger, how many pounds of chicken can you buy? Show your work.

f. If you buy 4 pounds of chicken, how many pounds of hamburger can you buy? Show your work.

7. Mr. Banks is worried that he was overcharged by the automotive detailing shop when they cleaned his Mustang. He dropped his car off at 9 am and picked it up at 11 am. The worker charges $35 per hour together with a one-time $20 supplies fee.

a. Use the verbal model to help you write an equation that models total cost of having a car detailed.

\[
\text{Total cost} = (\text{Cost per hour}) \cdot (\text{Number of hours}) + \text{One-time supply fee}
\]
b. What are the independent and dependent variables?
c. Is this equation in standard form or slope-intercept form? Explain how you can tell.

d. If the bill came to $125 many hours did Mr. Banks pay for? (Use your equation to answer this question.)

e. Should Mr. Banks speak to the manager?

8. André and Teresa ride their mountain bikes along the old logging roads in the Berkshires where cell phone reception is unreliable. Their walkie-talkies have a range of 60 miles. From their starting point, they decide to explore in opposite directions, but they want to stop when they are out of range. When she is biking, Teresa is able to maintain a speed of 6 mph. André bikes at 5 miles per hour. Sometimes Teresa or André will stop and take a rest, swim in the stream, eat lunch, or fix a flat tire. How many hours could Teresa and André be riding on their bikes before they lose touch with each other?

a. Use the verbal model below to help you write an equation that models the different number of hours that Teresa and André might be on their bikes.

\[
\text{Distance Teresa travels} + \text{Distance André travels} = \text{Range of walkie-talkie}
\]

\[
(Teresa’s rate \cdot Teresa time on bike) + (André’s rate \cdot André’s time on bike) = \text{Range of walkie-talkie}
\]

Let \( T \) be the total hours that Teresa is actually biking.

Let \( A \) be the total number of hours that André is on his bike.

Write an equation that models the situation.

b. Is this equation in standard form or slope-intercept form? Explain how you can tell.
c. Find the $T$-intercept. What does it mean in the context of the problem?

d. Find the $A$-intercept. What does it mean in the context of the problem?

e. Graph using the intercepts. Make sure to label the axes and choose a proper scale.

g. If André bikes for 7.2 hours, how long can Teresa ride? Show your work.
Practice with Standard Form and Slope-Intercept Form

1. In its most recent game, the North High School girls’ basketball team scored 72 points not including foul shots. A sports writer wonders how many 3 pointers and how many 2 pointers could have been made.

   a. Let $x$ represent the number of 2-point goals and $y$ the number of 3-point goals. Make a table showing possible combinations that give a total of 72 points.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

   b. Plot these ordered pairs on a graph.

   ![Graph](image)

   c. Draw a line through the points you have plotted.

   d. What is an equation for the line?
e. What is the $x$-intercept? What is the meaning of the $x$-intercept?

f. What is the $y$-intercept? What is the meaning of the $y$-intercept?

g. What is the slope? What is the meaning of the slope?

h. Does every point on the line represent a possible combination of 2-point and 3-point goals? Explain.

i. Rewrite this equation in slope-intercept form.

j. Confirm that the slope ($m$) and $y$-intercept ($b$) in slope-intercept form are the same as you obtained earlier.
2. Mohammed and his four friends begin a long hike with 225 pounds of food. They plan to eat a total of 12 pounds of food per day.

   a. Define the variables and write an equation to show how the amount of food remaining is related to the number of days they have been on the hike.

   b. What is the y-intercept and what does it mean in this situation?

   c. What is the slope and what does it mean in this situation?

   d. Use the equation to determine when the hikers will run out of food.

3. The owner of a candy store decides to make a mixture of nuts to sell. He uses peanuts and cashews. He would like the mixture to weigh five pounds and needs to know what possible combinations he can use.

   a. Let \( x \) represent the weight of the peanuts and \( y \) represent the weight of the cashews. Make a table showing possible combinations of nuts.

   \[
   \begin{array}{|c|c|}
   \hline
   x & y \\
   \hline
   \hline
   \hline
   \hline
   \hline
   \end{array}
   \]
b. Plot these ordered pairs on a graph.

c. Draw a line through the points you have plotted.

d. Write an equation of the line.

e. Use the equation to fill in the table and find the $x$ and $y$ intercepts.

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & 0 \\
0 & \\
\hline
\end{array}
\]

f. What is the $x$-intercept? What is the meaning of the $x$-intercept?

g. What is the $y$-intercept? What is the meaning of the $y$-intercept?
h. What is the slope and what does it mean in this situation?

i. Does every point on the line between the two intercepts represent a possible combination of peanuts and cashews? Explain.

j. Rewrite the equation of this line in slope-intercept form.
Unit 4: Investigation 6 (4 Days)

POINT-SLOPE FORM OF LINEAR EQUATIONS

CCSS: F-LE5, F-LE2, F-IF8, F-LE1

Overview
Students learn to use the point-slope form of a linear equation and develop a deeper understanding of functions as they solve a variety of contextual problems. Students discover that the slope-intercept, point-slope and standard forms of a linear equation are equivalent, and students learn to select a form that best fits the data or the question to be answered.

Assessment Activities

Evidence of Success: What Will Students Be Able to Do?
- Write an equation of a line in the context of a real world problem.
- Write the equation of a line in slope-intercept form, point-slope form, or standard form given (1) the slope and y-intercept, (2) the slope and one ordered pair on the line, (3) two ordered pairs or (4) an ordered pair and an equation of a parallel or perpendicular line.
- Transform an equation from slope-intercept form or point-slope form to standard form.
- Transform an equation from point-slope form or standard form to slope-intercept form.
- Make predictions based on the meaning of the function.
- Use slope and intercepts to analyze real world problems.

Assessment Strategies: How Will They Show What They Know?
Exit Slip 4.6.1 assesses fundamental understanding of the point-slope form by asking students to identify the slope and the point from an equation and to write an equation given the slope and one point.
Journal Entry 1 asks students to explain the meaning of point-slope form.
Exit Slip 4.6.2 has students apply point-slope form in a real world context.
Journal Entry 2 asks students to evaluate advantages and disadvantages of point-slope form.

Launch Notes
Open this investigation with a discussion about the issue of bottled water vs. tap water, which was introduced in Unit 3. To prepare for this discussion the teacher may wish to ask students to visit web sites on the issue as homework the previous day. A current news story may be used to introduce the assignment. For example, in May 2009 the governor of New York banned state agencies from purchasing bottled water. Ivy League colleges are distributing reusable water bottles and creating public hydrating stations. For more information search “bottled water ban in….” or the New York Times archives on bottled water:
You could facilitate a class debate or small group discussions. Reasons favoring bottled water include convenience, taste, and possible water quality. Reasons favoring tap water include cost and the problem of recycling plastic bottles.

**Closure Notes**

By the end of this investigation, students should have facility with the three forms of linear equations: slope-intercept, standard and point-slope. They should be able to articulate the benefits of each of the three forms, choose among them according to the dictates of the problem at hand, and recognize the mathematical equivalence of an equation in the three forms. Tell students to focus on finding the slope and a point if they want to find an equation of a line.

**Teaching Strategies**

1. Following the discussion, have students work through Activity 4.6.1 Trends in Bottled Water Consumption which provides a review of previous work with linear functions, practice with general numeracy and estimation, and introduces the point-slope form of the line.

   The last form of the linear equation to be studied is the point-slope form. Introduce the key idea that the slope between any point and a fixed point on a line will be constant. Using the formula for slope and the characterization of a linear function as having a constant rate of change, guide the students to discover the point-slope form of the line. Students should recognize that since the slope is the same between the point \((x_1, y_1)\) and any other point \((x, y)\) on the line, then \((y - y_1)/(x - x_1) = m\) where \(x\) and \(y\) vary, but \(m\) is a constant. Multiplying both sides of the equation by the denominator will give the point slope form: \(y - y_1 = m(x - x_1)\). Students should find any ordered pair \((x, y)\) by substituting a value of \(x\) and solving for \(y\). Have students use a point and the slope from the point-slope form to graph the equation. Students should note that when the given point is \((0, b)\) the point-slope of the equation becomes \(y - b = m(x - 0)\) and thus \(y = mx + b\). While the slope-intercept form highlights the slope and \(y\)-intercept, the point-slope form highlights ANY point on the line we want to draw attention to and the slope.

   Students then have numerous opportunities in Activity 4.6.2 Point-Slope Form of an Equation to explore the point-slope form in greater depth. You may use Exit Slip 4.6.1 Point-Slope Form to check for basic understanding of the point-slope form.

   Although we want to emphasize writing equations to model real world problems, some of the problems can be solved without writing equations. Since this lesson is about writing equations, it is important to do so, but teachers should acknowledge the thinking process of students who choose other approaches.
Differentiated Instruction (For Learners Needing More Help)

Add to the arsenal of the students’ notecards by filling out cards for point-slope form: how to read a point and the slope from an equation in point slope, how to write the equation of a line given a point and the slope, how to transform equations from one form to another.

Assign students to update the bulletin board.

Differentiated Instruction (Enrichment)

Students may research water facts and create word problems for the class to solve with the data they uncover. A group may decide to collect empty water bottles school wide. Have them research how to recycle water bottles and caps. Forest Elementary students made a green house out of empty water bottles. http://www.hometownlife.com/article/20120603/NEWS06/206030372/Forest-students-recycle-water-bottles-make-greenhouse

As the students collect water bottles, they are collecting data that may be used in a future math investigation.

Journal Prompt 1

A wise person once said that to get somewhere, you need to know where you are starting from and how to move. How is this like graphing a line in point-slope form?

II. **Activity 4.6.3 Practice with Point-Slope Form** provides students additional exercises on point-slope form. Guide students toward the idea that all they need to determine an equation of a line is a point and the slope. If the slope is missing, they should look for a hint as to how to find it. For example, they may be given two points to which they can apply the slope formula, or they may be given a rate or a line parallel to the given line from which they can infer the correct slope. If a point is missing, sometimes it is disguised as a starting value or a fixed cost. If the directions give the x-intercept, students may recall that the x-intercept is indeed a point in the form \((c,0)\). Contextual problems involving point-slope form are interspersed throughout the activities. The wind chill application in Exit Slip 4.6.2 Wind Chill provides an opportunity to assess students’ ability to create a linear function.

When students ask the question “Does it matter which point I use when finding the equation of a line between two points?” have them do **Activity 4.6.4 Can We Both Be Right?**

III. As needed, provide more practice transforming equations from point-slope form and standard form to slope-intercept form and from standard form and point-slope...
form to slope-intercept form by assigning Activity 4.6.5 Transforming Linear Forms.

**Group Activity**

Make up several sets of cards, one set for each of several functions. Each set will consist of 5 cards for a given function: Card 1 has the word problem on it. Card 2 has the corresponding function in standard form. Card 3 has the graph of the function. Card 4 has the function in slope-intercept form and Card 5 has the function in point-slope form. If you have 30 students you will need 6 sets of 5 cards. Randomly distribute one card to each student. Have the students walk around the room trying to find the other 4 people that are mathematically equivalent to their function. When they have found their matches have each group report back to the class and explain why their cards match.

IV. Students should identify the advantages of using each form of the equation of the line. Lead a class discussion on which form to use in the problems in Activity 4.6.6 Finding and Using Linear Functions. In Problem 4 of this activity, students are given two points and asked to write equations in at least two of the three forms.

Have students complete a journal entry to assess student understanding of the advantages of each form of a linear equation and how to discern which relevant information is given in a real life problem and which form(s) of the equation to use. Activity 4.6.7 You Choose can be used in class for group work or homework. The water theme is continued in some of the problems.

Additional homework may include problems in which students practice the skills of finding equations for lines (a) given slope and $y$-intercept, (b) given one point and the slope, (c) given two points, and (d) given one point and a parallel or perpendicular line.

**Journal Prompt 2**

State the relative advantages and disadvantages of the point-slope form of an equation compared with the slope-intercept and standard forms.

**Resources and Materials**

- Activity 4.6.1 Trends in Bottled Water Consumption
- Activity 4.6.2 Point-Slope Form of an Equation
- Activity 4.6.3 Practice with Point-Slope Form
- Activity 4.6.4 Can We Both Be Right?
- Activity 4.6.5 Transforming Linear Forms
• **Activity 4.6.6** Finding and Using Linear Functions
• **Activity 4.6.7** You Choose
• **Exit Slip 4.6.1** Point-Slope Form
• **Exit Slip 4.6.2** Wind Chill
• Rulers
• Bulletin Board for key concepts
• Student Journals
Trends in Bottled Water Consumption

Here is a data table that shows the consumption of bottled water in the United States in the years 2000 and 2007 in billions of gallons. Let’s assume that during this period consumption was a linear function of time.

<table>
<thead>
<tr>
<th>Year</th>
<th>Billions of Gallons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>4.7</td>
</tr>
<tr>
<td>2007</td>
<td>8.8</td>
</tr>
</tbody>
</table>

1. Let \( x \) represent the number of years from 2000 and \( y \) represent the amount of water consumed in billions of gallons. Make a graph with \( x \) on the horizontal axis and \( y \) on the vertical axis by plotting the two points and using a ruler to draw a line between the two points (do not extend the line.)

2. Find the rate of change in water consumption per year using data for the years 2000 and 2007.

3. Use the rate of change from question 2 and the \( y \)-intercept of your graph to write a linear equation in slope-intercept form.
4. Use your equation from question 3 to determine the consumption of bottled water in 2004.

5. A more accurate figure for the consumption of bottled water in 2007 is 8757.4 million.
   a. Write this large number in standard decimal notation.
   b. Assume that the average water bottle contains 24 ounces. Estimate the actual number of water bottles sold. Use the fact that one gallon contains 128 ounces.
   c. The population of the United States in 2007 was about 300 million. On average how many water bottles were purchased by each person in the country?
   d. In 2008, bottled water consumption decreased from about 8.8 billion gallons in 2007 to 8.7 billion in 2008. What are some possible reasons?
6. Here is a table that shows the consumption of bottled water in the United States from 2007 to 2009 in billions of gallons. As you can see, consumption continues to decline. So from 2007 to 2009 we will use a new linear function to model bottled water consumption.

<table>
<thead>
<tr>
<th>Year</th>
<th>Billions of Gallons</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>8.80</td>
</tr>
<tr>
<td>2009</td>
<td>8.45</td>
</tr>
</tbody>
</table>

a. As you did in question 1 above, let \( x \) represent the number of years from 2000 and \( y \) represent the amount of water consumed in billions of gallons. Make a graph with \( x \) on the horizontal axis and \( y \) on the vertical axis by plotting the two points and using a ruler to draw a line between the two points and extending the line to the right.

b. Find the rate of change in water consumption per year using data for the years 2007 and 2009.

Unlike the equation for the water consumption in question 1, we are not able to find the \( y \)-intercept unless we do a little work. So how will we find an equation? Recall that the slope between any two points on a line is the same.

c. If water consumption continues to decline at a steady rate, the slope of the line will continue to be _______________.

Activity 4.6.1

CT Algebra I Model Curriculum Version 3.0
d. Use slope to count over to a third point on your line to the left of (7,8.8), and state the coordinates.

e. Then find the slope between the point you picked and the point (7,8.8).

Check with a few classmates to see if you all are getting the same slope you found in question 6b.

f. Now label an arbitrary point on your line (x,y). Instead of using numbers in the second point, use the variables x and y, because there are many different values of x and y on the line.

g. Use the slope formula to find the slope from this arbitrary point (x,y) to the point (7, 8.8) and you should still get the same slope. Why?

Here is the algebra:

\[
\frac{y-8.8}{x-7} = m \\
y - 8.8 = m(x - 7) \\
y - 8.8 = -0.175(x - 7)
\]

Do a little algebra to transform the equation.

Multiply both sides of the equation by x – 7 to bring the denominator up to the right side.

Fill in the value for slope and you will have an equation in Point-Slope form.

Point-slope form simply asserts that the slope between the fixed point (7, 8.8) on the line and any arbitrary point (x,y) is the same slope m; in this case m = -.175.

h. To confirm that this equation contains the point (7, 8.8), substitute x = 7 and y = 8.8 into the equation y – 8.8 = -0.175(x – 7). Check to see if the result is a true statement.

i. To confirm that this equation contains the point (9, 8.45), substitute x = 9 and y = 8.45 into the equation y – 8.8 = -0.175(x – 7). Check to see if the result is a true statement.

j. Solve for y to transform the equation to slope-intercept (y = mx + b) form.
Point-Slope Form of an Equation

1. Graph the equation \( y = \frac{3}{4}x + 4 \) by starting at (0,4) and moving to another point on the line using the slope.

2. Now draw another graph of \( y = \frac{3}{4}x + 4 \). This time pick the point (-8, -2) which is a point on the line, and use slope to count up and right from that point to find other points on the graph. Do you end up with the same line as you did in part 1a above?

3. Notice that you can find points on a line or graph a line by starting at a point and moving according to the slope. Does it matter which point on the line is chosen to start with?
Facts about Point-Slope Form

The point-slope form of a line is a special form that tells you the SLOPE of a line and one POINT on the line.

The point-slope formula is: \( y - y_1 = m(x - x_1) \)

- \( m \) is the slope of the line; it will be a number
- \( x_1 \) is the \( x \) coordinate of a particular point on the line, and it will be a number
- \( y_1 \) is the \( y \) coordinate of a particular point on the line, and it will be a number
- \( x \) is the variable \( x \)
- \( y \) is the variable \( y \)

Point-slope form comes from the fact that the slope between any two points on a line is always the same. Use the slope formula between the specific fixed point \((x_1, y_1)\) and any moveable point \((x, y)\):

\[
\frac{y - y_1}{x - x_1} = m
\]

Multiply both sides of this equation by the denominator \( x - x_1 \) to obtain: \( y - y_1 = m(x - x_1) \)

When you substitute the values of the specific point \((x_1, y_1)\) into the point slope formula, you will obtain a true statement “0=0” which proves that \((x_1, y_1)\) is a point on the line.

The slope-intercept form of a line is \( y = mx + b \). \( m \) is the same value in both forms. Notice that it is the coefficient of the \( x \) variable.

**EXAMPLE:** For the equation \( y - 7 = \frac{4}{9}(x - 2) \), the particular point is (2,7). The

- \( x \)-coordinate of this point is 2 and the \( y \)-coordinate of this point is 7. The line has slope of \( \frac{4}{9} \).

4. Verify that (2,7) is a solution to the equation \( y - 7 = \frac{4}{9}(x - 2) \) by evaluating the equation when \( x=2 \) and \( y=7 \).
5. For each equation in point-slope form, identify the particular point and the slope. Then graph each equation. Test your point in the equation to be sure that the point makes the equation true.

   a. \( y - 3 = \frac{2}{5}(x - 1) \)

   b. \( y - 8 = -\frac{3}{2}(x - 2) \)
c. \[ y - 2 = \frac{1}{5}(x - (-5)) \]

d. \[ y + 6 = \frac{1}{2}(x + 2) \]
6. Use the point and the slope to write an equation of the line in point-slope form.

\[ y - y_1 = m(x - x_1) \]

a. \((3,5), \ m = 2\)

b. \((2,6), \ m = \frac{-4}{7}\)

c. \((3,0), \text{ parallel to the line } y = 9x + 5\)

d. \((0,4), \text{ perpendicular to the line } y = \frac{-8}{5}x + 2\)

e. \((3,2), \ m = 0\)

f. \((-3,2), \ m = \frac{7}{5}\)

g. \((-5, -1), \ m = 3\)
7. Plot the two points (−3,7) and (1,−3).

   • Draw the line containing the two points.
   • What is the slope between the two points?
   • Write an equation in point-slope form:

8. Find an equation of the line between the given two points by first finding the slope, then finding the point-slope form of the equation.

   a. through points (5,8) and (−2,7)

   b. through points (−2,−6) and (−7,5)
9. This February and March, the middle school students had their most successful food drive, topping last year’s total by 57 items. They started the food drive on day 0 with 8 cans of fruit juice which had been donated too late to be included in the November food drive. Contributions poured in at a constant rate of 12 food items per day. By the time the drive was over, the cans covered the cafeteria stage.

a. What is the dependent variable?

b. What is the independent variable?

c. Find the slope described in the situation.

d. Find a point described in the situation.

e. Write an equation of the line in point-slope form.

f. Write an equation of the line in slope-intercept form.

g. Use either equation to tell how many food items had been collected by the 10th day.

h. Use either equation to tell how many days it took to collect 488 items.
Transforming a function from point-slope form into slope-intercept form

10. a. Sketch the graph of the function $y - 2 = 4(x - 3)$.

b. Transform the previous equation into slope intercept form by applying the distributive property on the right side and solving for $y$.

c. What are the slope and $y$-intercept?

d. Confirm that the equation in slope-intercept form gives the same graph as the equation in point-slope form.
11. a. Sketch the graph of the function \( y + 4 = \frac{3}{4}(x + 6) \).

b. Transform the previous equation into slope-intercept form by applying the distributive property on the right side and solving for \( y \).

c. What are the slope and \( y \)-intercept?

d. Confirm that the equation in slope-intercept form gives the same graph as the equation in point-slope form.
Practice with Point-Slope Form

1. Is \((-7,5)\) a point on the graph of the line \(y - 5 = \frac{9}{4}(x + 7)\)? (Hint: Evaluate the equation when \(x = -7\) and \(y = 5\).) Show your work.

For each equation in point-slope form, identify the point and the slope. Then graph each equation. Test your point in the equation to be sure that the point makes the equation true.

2. \(y - 6 = \frac{-3}{2}(x - 1)\)
   
   point is (___,____) , slope is ____
   
   Substitute your point into the equation and simplify:

3. \(y - 2 = \frac{1}{5}(x + 7)\)
   
   point is (___,____) , slope is ____
   
   Substitute your point into the equation and simplify:
4. Find an equation for each line described below:

a. Through (10,6) with slope $\frac{-6}{5}$

b. Through (−3, −2) with slope 4

c. Through the two points (−1,2) and (3,4)
   
   Step 1: find the slope, $m$:

   Step 2: use the point-slope form to write an equation.

d. Through (7,−8) and parallel to the line $y - 4 = \frac{2}{5}(x - 3)$

e. $y$-intercept 8, slope $-4$
5. During recent flooding along the Connecticut River, water was rising at the rate of 4 inches per hour. At 3 AM the water level was 6 feet above normal.

a. Find an equation that gives the level of the water at any time during the day. (Let midnight = 0, noon = 12, 1 PM = 13, etc.).

b. When will the water reach the level of 12 feet above normal?

6. At 2 AM the river at the Harborside in Middletown was 1 foot above flood stage. By 5 AM the water had risen to 2.7 feet above flood stage.

a. What is the slope and what does it mean in this context?

b. Write an equation that gives the level of the water above flood stage at any time during the day. Let midnight be time 0.

c. Transform the equation from part (b) into slope-intercept form to find the y-intercept.

d. What is the y-intercept and what does it mean in this situation?
7. a. Sketch the graph of the equation: $y - 0 = 3(x - 2)$

b. Transform the equation into slope-intercept form.

c. What is the $y$-intercept?

8. a. Sketch the graph of the equation: $y - 7 = -4(x - 1)$.

b. Transform the equation into slope-intercept form.

c. What is the $y$-intercept?
Can We Both Be Right?

Alexis and Shonda are given the following problem to solve:

*Find an equation for the line passing through the two points (2, 7) and (5, 13).*

Alexis says, “I know, the first thing we can do is to find the slope.” Shonda agrees. They calculate the slope and get the same answer.

1. What is the slope of the line?

Shonda says, “Now that we have the slope, we can use the point-slope form of the equation.”
Alexis agrees, but asks, “Which point should we use?”
“I’m not sure,” replies Shonda. “Does it really matter?”
“Let’s each try it with a different point and see if we get the same answer,” Alexis suggests.

2. Alexis uses the point (2, 7) to find an equation. What equation does Alexis find?

3. Shonda uses the point (5, 13) to find an equation. What equation does Shonda find?

“These two equations sure look different,” Shonda comments. “I wonder if one of us made a mistake.”

4. Show that your two equations are indeed the same by rearranging both into slope-intercept form.
Can We Both Be Right?

Find an equation for the line passing through the two points \((6, 9)\) and \((12, 13)\).

Alexis says, “I know, the first thing we can do is to find the slope.” Shonda agrees. They calculate the slope and come up with the same answer.

1. What is the slope of the line?

Shonda says, “Now that we have the slope, we can use the point-slope form of the equation.”
Alexis agrees, but asks, “Which point should we use?”
“I’m not sure,” replies Shonda. “Does it really matter?”
“Let’s each try it with a different point and see if we get the same answer,” Alexis suggests.

2. Alexis uses the point \((6, 9)\) to write an equation in point-slope form. What equation does Alexis write?

3. Shonda uses the point \((12, 13)\) to write an equation in point-slope form. What equation does Shonda write?

“These two equations sure look different,” Shonda comments. “I wonder if one of us made a mistake.”

4. Show that your two equations are indeed the same by rearranging both into slope-intercept form.
Transforming Linear Forms

<table>
<thead>
<tr>
<th>Slope-intercept form: $y = mx + b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard form: $Ax + By = C$</td>
</tr>
<tr>
<td>Point-slope form: $y - y_1 = m(x - x_1)$</td>
</tr>
</tbody>
</table>

1. Transform each equation into standard form:
   
   a. $y = 2x - 8$

   b. $y = \frac{-4}{7}x + 2$

   c. $y - 2 = -5(x + 1)$

   d. $y + 4 = \frac{2}{5}(x + 3)$
2. Transform each equation into slope-intercept form:

   a. $4x - 7y = 21$

   b. $3x + 5y = 8$

   c. $y - 2 = -5(x + 1)$

   d. $y + 4 = \frac{2}{5}(x + 3)$
Finding and Using Linear Functions

To do these problems, choose among the three forms of linear equations we have studied:

- **Slope-Intercept Form:** \( y = mx + b \)
- **Standard Form:** \( Ax + By = C \)
- **Point-Slope Form:** \( y - y_1 = m(x - x_1) \)

1. **Pedro’s Parking Ticket**

   Pedro thought he could just run into the store for a minute, so he didn’t put any money in the parking meter. He got a ticket for $25 due one week later. The ticket said that it would be an additional $13.00 for each day it was paid late.

   a. Define the variables and write an equation that represents the total fine for any number of days late paying the ticket.

   b. Use the equation to find the amount Pedro must pay if he is six days late.

   c. When Pedro finally goes to the Town Clerk to pay the bill he has to pay $155.00. How many days late was he?
2. **Tahira’s Taxicab Ride**

In Boston taxicabs charge an initial fare and an additional amount for every mile traveled. Tahira is having trouble figuring out how the system works. She paid $7.00 for a two-mile ride one day and she paid $24.50 for a nine-mile ride the next day.

a. What is the independent variable? What is the dependent variable?

b. Use the data given to find the amount the taxis charge per mile.

c. Write an equation relating the two variables.

d. Use the equation to find how much a 15-mile taxicab ride would cost.

e. What is the slope and what does it mean in this situation?

f. What is the initial price Tahira has to pay before she has traveled anywhere? Explain.
3. **Car Wash**

The Outdoor Adventure Club at Eisenhower High School needs to raise money for their trip to Mountain Classroom, so they plan several fund raising events. The first one is a car wash. The total cost of sponges, soap, and other materials was donated by a local car dealership. They plan to charge $4.50 for each small car they wash and $8 for each large car or SUV. In the end they raised $420 for their trip.

a. Define the variables and write an equation to show how the money they raised is related to the two types of cars that they washed.

b. What is the $y$-intercept and what does it mean in this situation?

c. What is the $x$-intercept and what does it mean in this situation?

d. What is the slope and what does it mean in this situation?
4. Writing Equations

Write equations in point-slope form, slope-intercept form, or standard form for the line that passes through each pair of points. Try to use at least two forms of the equation for each pair.

<table>
<thead>
<tr>
<th>Pair of Points</th>
<th>Point-slope form</th>
<th>Slope-intercept form</th>
<th>Standard form</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. (5, 3) and (7, 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. (0, 8) and (3, 0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. (5, -3) and (-3, -4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. (-1, 2) and (0, 6)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e. (4, 9) and (0, 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
You Choose

This activity contains several word problems. Decide which form of a linear equation you wish to use to model each situation and then answer the questions.

1. **Kicking the Habit.** Carlos has been in the habit of buying a 20-ounce bottle of water at a convenience store on his way home from school every day. Each bottle costs $1.29. One of his friends tells him that at the sporting goods store in the mall there is a sale on reusable stainless steel bottles. The regular price is $29.95 but he can get one for $19.95. He can fill the reusable bottle every day with tap water, which tastes just as good as bottled water, carry it in his backpack and enjoy a cool drink at the end of the school day. Carlos doesn’t mind the extra weight of carrying a full bottle with him and he believes he can save some serious money, so he goes ahead and buys the reusable bottle. He wonders how much he will actually save. For the advantages of stainless steel water bottles, visit [http://www.thegoodhuman.com/2008/03/17/choosing-a-safe-reusable-water-bottle/](http://www.thegoodhuman.com/2008/03/17/choosing-a-safe-reusable-water-bottle/)

   a. Let \( d \) represent the number of school days after buying the reusable bottle and \( s \) represent the amount of money saved. Write an equation for \( s \) as a function of \( d \).

   b. In your equation, what is the slope? What does it represent in the context of the problem?

   c. In your equation, what is the \( s \)-intercept? What does it represent in the context of the problem?

   d. How much will Carlos save at the end of six weeks?

   e. On what day will Carlos break even?
2. **Brittany’s Hot Tub.** When draining her hot tub, the height of the water in Brittany’s hot tub decreases at a rate of 2 inches per minute. After 9 minutes there are 11 inches of water in the tub. Assume that the height is decreasing at a constant rate.

a. Write an equation for the height of water left in the tub \( h \) as a function of the number of the minutes \( t \) that have elapsed.

b. How high is the tub when it is full?

c. After how many minutes will the tub be empty?

d. What values of the domain, \( t \), make sense? Why?

e. What values of the range, \( h \), make sense? Why?
3. **The Water Cooler.** Ms. Robinson is a guidance counselor at North High School. She has a water cooler in her office so that when students visit they may help themselves to a drink. The water cooler uses large bottles that are delivered by a bottling service. The service delivers a new bottle every other Monday.

One Monday morning a newly filled water bottle is delivered to her office. On Tuesday morning Ms. Robinson measures the level of the water in the tank and finds that it is 12.7 inches high. Three days later, on Friday morning, the level is 8.2 inches high. She wonders whether she will run out of water before the next delivery. Assume that the number of days since the bottle was delivered is a linear function of the height of the water.

a. Find the slope of the line representing the height of the water as a function of the number of school days that pass since each delivery.

b. Define the variables and write an equation for the line.

c. Predict the level of the water on the following Tuesday morning.

d. When will there be only 1 inch of water left in the bottle?

e. How high was the water level when the bottle was delivered full?

f. The bottling service plans to deliver water bottles every two weeks. Will this plan meet the needs of Ms. Robinson and her students? Explain.

g. Water bottles for this cooler are cylindrical in shape. They are 10 inches in diameter and 15 inches high. What is the volume of the bottle in gallons? Use the fact that one gallon contains 231 cubic inches.
4. **Water Boy.** Steve Ballmer, the current CEO of Microsoft, used to be the manager of his college football team. Among his duties, he had to be sure the players were hydrated. When nearby construction forced a water shut off, Steve went to the Star Market to purchase bottles of water. He needed a total of 80 liters of water. Star Market sold water in two liter bottles and in half liter bottles. What possible combinations of the small and large bottles might he purchase in order to bring 80 liters to the football team?

a. Write an equation that models the possible combinations of half liter bottles and two liter bottles that would total 80 liters. (Be sure to define the variables.)

b. What is the $x$-intercept and what does it represent?

c. What is the $y$-intercept and what does it represent?

d. If the store only had 48 half-liter bottles, how many 2 liter bottles should Steve purchase?